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1. Prove that a space X is connected iff it contains no nonempty proper subset which is both open and closed. (A is a **proper** subset of X iff $A \subset X$ but $A \neq X$.)
 2. True or false: If A and B are connected subspaces of a topological space X and $A \cap B \neq \emptyset$, then $A \cup B$ is connected. Prove your answer.
 3. True or false: If A and B are connected subspaces of a topological space X and $A \cap B \neq \emptyset$, then $A \cup B$ is connected. Prove your answer.
 4. Prove the following.
 - (a) Theorem: The continuous image of a connected set is connected; i.e, if $f : X \rightarrow Y$ is a continuous map between topological spaces and X is connected, then $f(X)$ is connected.
 - (b) Corollary: If X is connected, and Y is homeomorphic to X , then Y is connected.
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Extra Credit Problems

5. Prove the following theorem: $A \subset \mathbb{R}$ is connected iff A is an interval (open, closed, or half open; infinite or half-infinite).
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