Midterm #2. Math 104, Symmetry, Shape, Space.
Instructor: Ramin Naimi

Name: _____
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Closed book. Closed Notes. No Calculators. Do **only four** of the following problems. 25 points per problem. Please write very legibly.

1. (a) Use a compass and a straightedge to draw the diagram described below:
   AB is a given line segment of unit length. $ABCD$ is a square with AB as a side. F is the midpoint of AB. E is a point where the continuation of line AB intersects the circle that has F as its center and AC as its radius. Show your work (the various "marks" you make); no explanation needed for each step.
   (b) Prove that the length of AE is the golden ratio, $\phi = (1 + \sqrt{5})/2$.
   (c) Use the above to draw a golden triangle with a compass and a straightedge. Explain briefly.
2. What is the relationship between a regular pentagon and the golden ratio $\phi = (1+\sqrt{5})/2$? Explain and prove your answer.
3. Describe in detail how one can use Origami Postulates 1-5 to copy a given angle $\angle ABC$ at the point D on a given line segment DE.
4. (a) Suppose AB is a given line segment, and L is a line that intersects AB at some point between A and B. Explain how you can use Origami Postulate 5 to find a point C on L such that AB and BC have the same length.
   (b) Describe in detail how one can use Origami Postulates 1-5 to construct an equilateral triangle.
5. Consider a linkage made of two rods, AB and CD, where A is fixed, B is a joint at the midpoint of CD that's free to rotate about A, and C is restricted to move along a horizontal line through A. Describe the path that D follows as we move B. Prove your answer.
6. (a) What is a regular tiling? What is a semiregular tiling?
   (b) Can there be four different polygons at a vertex of a semiregular tiling? Why?
7. Suppose we place two mirrors at the two ends of a 12-inch ruler, so that they are perpendicular to the ruler. We draw a dot at the five-inch mark. What is the distance between the dot and its *second* image to the left of the first mirror? Explain why.
8. Describe what the Rosette Group for an equilateral triangle is. Make the multiplication table for this group, and explain *how* you construct it.