
Closed book. Closed Notes. 20 points per problem. Please write very legibly.

Do **only two** of the following problems.

Please circle the two problems you are choosing:

1.

2.

3.

1. Suppose we have a graph whose vertices are labeled A, B, C, D, E, F , and has the following edges: $AB, AC, AE, AF, BC, BD, CD, CE, DE, DF, EF$.
 - (a) What is the degree of each vertex?
 - (b) Does this graph have an Euler path? If yes, describe the path using the labeled vertices. If not, explain why not.
2.
 - (a) What is the relationship between the number of edges and the sum of all the degrees of the vertices in any graph? Briefly explain why.
 - (b) Explain why the number of odd degree vertices in any graph is always even.
3. In the Handshake problem, explain why Ben cannot have shaken four hands. Make sure you're not assuming anything more than what's given in the problem; you should explain and justify everything else. (Recall the problem: three couples, one of which is Alice and Ben; no one shakes their partner's hand; no two people shake each others' hands more than once; Alice gets five different answers when she asks everyone how many hands they shook.)