Closed book. Closed Notes. Please write very legibly. 20 points per problem.

1. Let the function f be defined for positive integers by

$$f(n) = \begin{cases} 3 & \text{if } n = 1\\ 0 & \text{if } n = 2\\ 2f(n-1) + f(n-2) & \text{otherwise} \end{cases}$$

Write the function f in Mathematica and use it to compute f(20). Show all work.

2. Define a function avg[] that computes the average (mean) value of its arguments; the function should work for any number of arguments ≥ 1 , regardless of whether the arguments are given in a list or as a sequence. Examples:

In[]: avg[6,7]
Out[]: 6.5
In[]: avg[{6,7}]
Out[]: 6.5
In[]: avg[6,7,8,9]
Out[]: 8.5
In[]: avg[{6}]
Out[]: 6

3. Use recursion to write a function $\mathtt{mult[m_,n_]}$ that, for integers $m \leq n$, gives the product $m(m+1)(m+2)\cdots n$. For example, $\mathtt{mult[2,4]}$ should give 24, since $2\cdot 3\cdot 4 = 24$, and $\mathtt{mult[5,5]}$ should give 5. Your function should work for negative integers as well. You may not use the built-in Mathematica function Factorial[].

4. Use recursion to write a function fitin[lis_,n_] that inserts n into a sorted list of numbers in the appropriate place, as in the following examples. Assume that lis is sorted in *descending order*, i.e., from large to small. You may not use the built-in Mathematica function Sort[].

In[]: fitin[{8,7,0,-2},3]
Out[]: {8,7,3,0,-2}
In[]: fitin[{8,7,0,-2},7]
Out[]: {8,7,7,0,-2}

5. Use recursion to write a function sortNumbers[lis_] that sorts any nonempty list lis of numbers in *descending order*. Even if you didn't do the previous problem (correctly), you may for this problem assume that the function fitin[] is given to you and use it. You may not use the built-in Mathematica function Sort[]. Example:

In[]: sortNumbers[{4,6,0,-9,3,2,6,2}]
Out[]: {6,6,4,3,2,2,0,-9}