Final Exam
CS 165, Mathematica.
Instructor: Ramin Naimi

Name: _____
Tuesday 4 May 2004

Closed book. Closed Notes. Please write very legibly. 20 points per problem.

1. (a) Use a `For[]` loop to write a function `makeList[n_]` that outputs the list `{1,2,3,...,n}`. Do not use the function `Table[]`.

   (b) Use two nested `For[]` loops to write a function `triangularMatrix[n_]` whose output is the same as the following function. Do not use the function `Table[]`.

   ```
   f[n_] := MatrixForm[Table[j, {i, 1, n}, {j, i, n}]];
   ```

2. Write a function `compareLists[lis1_,lis2_]` that outputs one of the letters `L`, `G`, or `N` according to the following rule (you may assume the two lists have equal length):

   1. If `lis1[[i]] < lis2[[i]]` for every `i`, then output = `L` (for "Less than"). 2. If `lis1[[i]] > lis2[[i]]` for every `i`, then output = `G` (for "Greater than"). 3. Otherwise output = `N` (for "Neither").

3. Recall the Collatz (or the $3n+1$) function: $f(n) = \begin{cases} 3n+1 & \text{if } n \text{ is odd} \\ n/2 & \text{if } n \text{ is even} \end{cases}$. Use *recursion* to write a function `collatzSeq[n_]` that gives the sequence of numbers one gets by starting with `n` and repeatedly applying the function f until the output becomes 1.

4. Write a function to find the smallest positive integer n such that $19n + 1$ and $59n + 1$ are both perfect squares. (Your answer should include the function as well as n.)

5. Rewrite the following function by replacing the anonymous function with an auxiliary function. The definition of `f` should not change except for replacing the anonymous function and adding the auxiliary function.

   ```
   f[x_] := Map[({#, #[[1]]/#[[2]]}) &, Table[{i, i + 1}, {i, 1, x}]]
   ```

6. (a) Write a function to plot $\tan(x) + 1000$ from $x = -2$ to $x = 2$?

   (b) The function $\tan(x) + 1000$ is drawn incorrectly by Mathematica. In one or two sentences explain the error in the plot and what the function should actually look like.

   (c) Another concern with the plot is that the axes do not cross at (0,0). Modify your function from part (a) so the axes cross at (0,0).

7. (a) Write a function `triples[n_]` that outputs a list of all integer triples $\{x, y, z\}$ with $1 \le x \le y \le z \le n$, as in the following example:

   ```
    In[]: triples[3]
   Out[]: {{1,1,1},{1,1,2},{1,1,3},{1,2,2},{1,2,3},
           {1,3,3},{2,2,2},{2,2,3},{2,3,3},{3,3,3}}
   ```

   (b) Use a replacement rule to write a function `pythagoreanTriples[n_]` that outputs a list of all integer triples x, y, z with $1 \le x \le y \le z \le n$ such that $x^2 + y^2 = z^2$. Example:

   ```
    In[]: pythagoreanTriples[14]
   Out[]: {{3,4,5},{5,12,13},{6,8,10}}
   ```