Mathematics 105: Math as a Liberal Art. Final Exam. Name_____
Instructor: Ramin Naimi Spring 2008
Close book. Closed notes. No Calculators . NO CELL PHONES! Please turn off your cell phones and put them away.

Solve the problem.

1. Which of the following four graphs is a tree?

Graph 1 Graph 2 Graph 3 Graph 4

a) Graph 2 and Graph 3
b) Graph 2 and Graph 4
c) Graph 1 and Graph 4
d) Graph 1 and Graph 3
e) Graph 3 and Graph 4
f) None of the above

2. A tree is
   a) any graph that is connected and has no circuits.
   b) any graph with that is connected and has no bridges.
   c) any graph that has at least one bridge.
   d) any graph that is not connected and has no bridges.
   e) any graph that has no circuits.
   f) None of the above

3. A tree is
   a) any graph with one component and no circuits.
   b) any graph for which there is one and only one path joining any two vertices.
   c) any connected graph whose edges number one fewer than its vertices.
   d) All of the above define a tree.
   e) None of the above

4. The number of edges in a tree with 49 vertices is
   a) 2^{49}.
   b) cannot be determined without further information.
   c) 49.
   d) 50.
   e) 48.
   f) A tree cannot have 49 vertices because every tree has an even number of vertices.

5. Which of the graphs described below must be a tree?
   a) An ancestry graph: People are the vertices; birthparents are joined to their offspring by edges.
   b) The students at Tasmania State University are the vertices; edges represent whether two students have had a class together or not.
   c) A minimal spanning subgraph of a weighted network representing cities and distances between them.
   d) All of the above graphs must be trees.
   e) Exactly two of the above graphs must be trees.
   f) None of the above graphs must be trees.

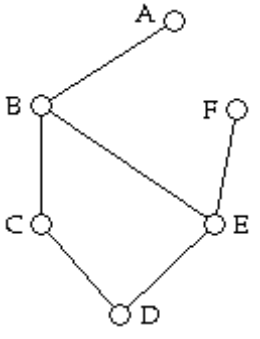
6. Assume that G is a graph with no loops or multiple edges. For each of the following, choose the option that best applies: (I) G is definitely a tree; (II) G is definitely not a tree; (III) G may or may not be a tree.
   a) G has 14 vertices, and they all have degree at least 2.
   b) G has 14 vertices; one vertex has degree 13 and the other vertices each have degree 1.
   c) G has 14 vertices, and every edge in G is a bridge.
   d) G has 14 vertices and is connected.
   e) G has 14 vertices, 13 edges, and no circuits.
   f) G has 14 vertices, and the degree of each vertex is either 1 or 2..

7. Suppose T is a tree with 41 vertices. Then
   a) T has 40 bridges.
   b) T has no bridges.
   c) T has one bridge.
   d) T can have any number of bridges.
   e) None of the above

8. Suppose G is a graph with 39 vertices and 38 edges. Then
   a) G must be a tree.
   b) G cannot have any circuits.
   c) G is either a tree or it is not connected.
   d) G cannot have more than one path joining any two vertices.
   e) None of the above

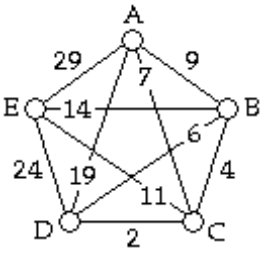
9. A graph has 37 vertices and has the property that there is one and only one path joining any two vertices of the graph. Which of the statements [A), B), C), or D)] is not true?
   a) Every edge of the graph must be a bridge.
   b) G must have 36 edges.
   c) G must be connected.
   d) G cannot have any circuits.
   e) All of the above statements are true.
   f) Only two of the above statements are true.

10. Every spanning tree of the following graph contains edge



   a) CD
   b) BC
   c) DE
   d) BE
   e) None of the above

The question(s) that follow refer to the problem of finding the minimum spanning tree for the weighted graph shown below.



11. Using Kruskal's algorithm, which edge should we choose second?
    a) BC
    b) BD
    c) AB
    d) AC
    e) CD
    f) None of the above

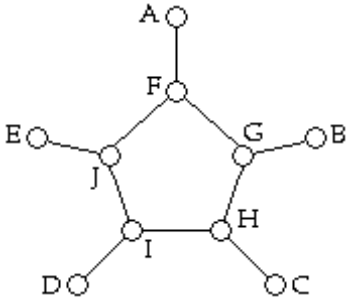
12. Using Kruskal's algorithm, which edge should we choose third?
    a) BD
    b) CD
    c) BC
    d) AC
    e) AB
    f) None of the above

13. Which of the following edges of the given graph are not part of the minimum spanning tree?
    a) CD
    b) AC
    c) BC
    d) BD
    e) EC
    f) None of the above

14. The total weight of the minimum spanning tree is
    a) 24.
    b) 13
    c) 20.
    d) 68.
    e) 26.
    f) 125.

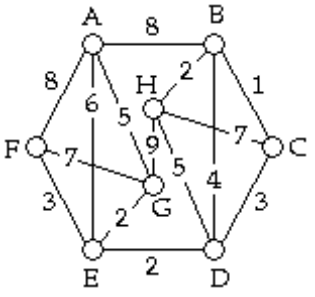
Solve the problem.

15. How many spanning trees does the following graph have?



a) 4
b) 6
c) 5
d) 7
e) None of the above

New telephone lines must be installed to connect 8 cities (A, B, C, D, E, F, G, and H). Due to government regulations, the only possible connections that are allowed are shown by the edges on the graph below. The numbers on the edges indicate the cost (in millions of dollars) of each possible connection. The telephone company wishes to connect the cities in the cheapest possible way.



16. Using Kruskal's algorithm, which edge is chosen last?
a) HG
b) BC
c) AB or AF
d) AG
e) BH, EG or DE
f) CD or EF

17. Using Kruskal's algorithm, which edge is chosen second to last?
a) AF
b) AB or AF
c) BC
d) CH or FG
e) BH, EG or DE
f) CD or EF

18. The number of bridges in the optimal solution to this problem is
a) 9.
b) 0.
c) 7.
d) 8.
e) None of the above

Use the mileage chart shown below to find the minimum spanning tree for the 5 cities of Boston, Buffalo, Chicago, Columbus, and Louisville.

	Boston	Buffalo	Chicago	Columbus	Louisville
Boston	*	446	963	735	941
Buffalo	446	*	522	326	532
Chicago	963	522	*	308	292
Columbus	735	326	308	*	209
Louisville	941	532	292	209	*

19. Using Kruskal's algorithm which edge is chosen third?
    a) Boston - Columbus.
    b) Columbus - Chicago
    c) Columbus - Louisville.
    d) Boston - Chicago.
    e) Buffalo - Columbus.

20. Which of the following edges is not in the minimum spanning tree?
    a) Boston - Buffalo.
    b) Buffalo - Columbus.
    c) Columbus - Louisville.
    d) Columbus - Chicago.
    e) All of the above are in the minimum spanning tree.

Solve the problem.

21. Which of the following statements is true about Kruskal's algorithm.
    a) It is an inefficient algorithm, and it never gives the minimum spanning tree.
    b) It is an inefficient algorithm, but it always gives the minimum spanning tree.
    c) It is an efficient algorithm, and it always gives the minimum spanning tree.
    d) It is an efficient algorithm, but it doesn't always give the minimum spanning tree.
    e) None of the above

22. Suppose a graph G has V vertices and E edges, such that $V = E + 1$. If G is definitely a tree or definitely not a tree, explain why; if it can be either, draw examples of both.

Solve the problem.

23. An election is held among four candidates (A, B, C, and D). Using a voting method we will call X, the winner of the election is candidate A. Due to an irregularity in the original procedures, a new election is required. Before the new election takes place, one of the voters changes his mind and moves A from third choice to second choice on his ballot. All other voters vote the same way they did in the original election. In the new election, still using voting method X, candidate D wins the election. Based on this information, we can say that voting method X violates the
    a) Condorcet criterion.
    b) monotonicity criterion.
    c) independence of irrelevant alternatives criterion.
    d) majority criterion.
    e) None of the above

24. An election is held among four candidates (A, B, C, and D). Using a voting method we will call X, the winner of the election is candidate A. Due to an irregularity in the original election, a re-election is required. All voters cast the same ballots as before except that one of the voters moves candidate D from third place up to second place on her ballot. In the recount, still using voting method X, candidate D wins the election. Based on this information, we can say that voting method X violates the
    a) majority criterion.
    b) independence of irrelevant alternatives criterion.
    c) Condorcet criterion.
    d) monotonicity criterion.
    e) None of the above.

Solve the problem.

25. An election is held among four candidates (A, B, C, and D). Using a voting method we will call X, the winner of the election is candidate A. Due to an irregularity in the original vote count, a recount is required. Before the recount takes place, candidate B drops out of the race. In the recount, still using voting method X, candidate D wins the election. Based on this information, we can say that voting method X violates the
    a) independence of irrelevant alternatives criterion.
    b) majority criterion.
    c) monotonicity criterion.
    d) Condorcet criterion.
    e) None of the above.

26. Arrow's Impossibility Theorem implies (before answering, read *all* choices carefully)
    a) that every voting method violates at least one of the four of the fairness criteria.
    b) that in every election, every voting method violates all of the four of the fairness criteria.
    c) that in every election, every voting method violates at least one of the four of the fairness criteria.
    d) that every voting method violates all of the four fairness criteria.
    e) that in every election, each of the voting methods produces a different winner.
    f) None of the above

27. "If there is a choice that has a more than half of the first-place votes in an election, then that choice should be the winner of the election." This fairness criterion is called the
    a) monotonicity criterion.
    b) independence of irrelevant alternatives criterion.
    c) majority criterion.
    d) Condorcet criterion.
    e) None of the above

28. Consider an election with 500 voters and 5 candidates. What is the smallest number of votes that a plurality candidate could have?
    a) 100
    b) 99
    c) 250
    d) 251
    e) 3
    f) 101
    g) 249

29. An election is held among four candidates (A, B, C, and D). Using a voting method we will call X, the winner of the election is candidate A. However, candidate D beats each other candidate in a head to head, pairwise comparison. Based on this information, we can say that voting method X violates the
    a) independence of irrelevant alternatives criterion.
    b) majority criterion.
    c) monotonicity criterion.
    d) Condorcet criterion.
    e) None of the above

For an election with four candidates (A, B, C, and D) we have the following preference schedule:

Number of Voters	7	7	5	8
1st choice	D	B	C	C
2nd choice	B	A	A	A
3rd choice	C	D	B	D
4th choice	A	C	D	B

30. In this election,
    a) there is no Condorcet candidate.
    b) D is a Condorcet candidate.
    c) B is a Condorcet candidate.
    d) C is a Condorcet candidate.
    e) A is a Condorcet candidate.
    f) every candidate is a Condorcet candidate.

31. Using the plurality method, which candidate wins the election?
    a) D
    b) A
    c) B
    d) None of the above
    e) C

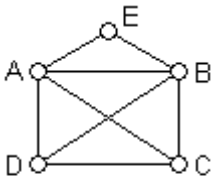
32. Write T (for True) or F (for False) next to each of the following.
    a) The Method of Pairwise Comparisons satisfies the Condorcet Criterion.
    b) The Borda Count Method satisfies the Majority Criterion.
    c) The Borda Count Method doesn't satisfy the Monotonicity Criterion.
    d) The Method of Pairwise Comparisons satisfies the Monotonicity Criterion.
    e) The Plurality Method doesn't satisfy the Majority Criterion.

For an election with four candidates (A, B, C, and D) we have the following preference schedule:

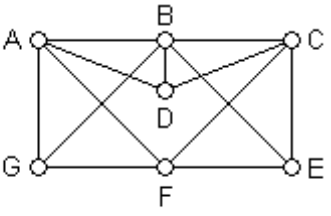
Number of voters	15	11	9	6	2
1st choice	A	C	D	B	C
2nd choice	B	B	C	D	D
3rd choice	C	D	B	C	B
4th choice	D	A	A	A	A

33. How many Borda points does candidate A earn in this election?
    a) 121
    b) 124
    c) 87
    d) 97
    e) 88

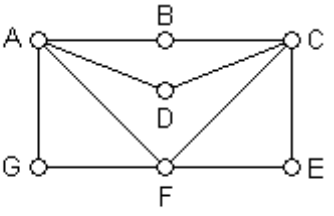
Use the figure below to answer the following question(s).



Graph 1



Graph 2



Graph 3

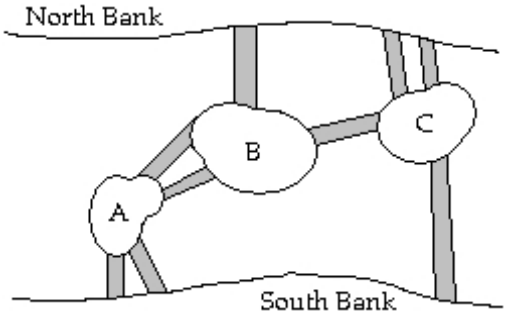
34. Which of the graphs has an open Euler path?
    a) Graph 2 only
    b) Graph 3 only
    c) Graphs 1 and 2
    d) Graph 1 only
    e) None of them.
    f) Graphs 1 and 3

Solve the problem.

35. A connected graph having all even vertices cannot have any bridges because
    a) vertices connected by a bridge must have odd degree.
    b) if all vertices are even then there are an even number of edges, which is impossible with bridges.
    c) if there are bridges, then Fleury's algorithm tells you there are no odd vertices.
    d) the number of odd vertices must be even, so there can be no bridges.
    e) a connected graph having all even vertices has an Euler circuit, which is impossible with bridges.
    f) a graph with bridges cannot be connected.

36. If a disconnected graph has exactly two odd vertices, then
    a) the two odd vertices must be in the same component of the graph because the graph has an open Euler path.
    b) the two odd vertices must be in the same component of the graph because each component must have an even number of odd vertices.
    c) the two odd vertices may be in the same component or in different components of the graph.
    d) the two odd vertices must be in different components of the graph because the graph has no Euler circuit.
    e) the two odd vertices must be in different components of the graph because the graph has no bridges.

In a certain city there is a river running through the middle of the city. There are three islands and nine bridges as shown in the figure below.



37. A graph that appropriately models this situation would have
    a) 5 vertices and 9 edges.
    b) 9 vertices and 5 edges.
    c) 3 vertices and 9 edges.
    d) 9 vertices and 3 edges.
    e) None of the above

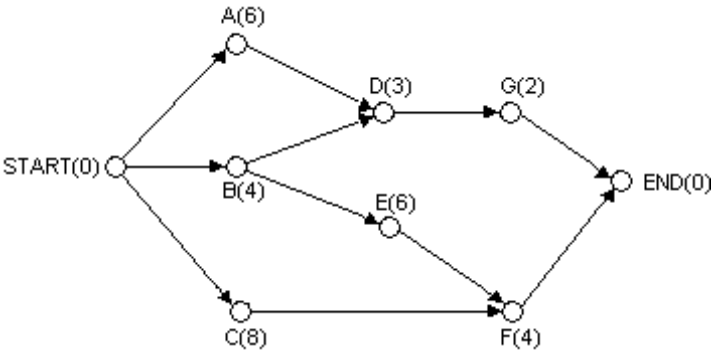
38. It is possible to take a walk though this town, starting on the South Bank, crossing each bridge once (and only once) and ending
    a) on island B.
    b) on the North Bank.
    c) on island C.
    d) on the South Bank.
    e) All of the above
    f) None of the above

39. Suppose there is a crossing charge of $1.00 every time one crosses a bridge. A tourist wants to start on the South Bank, stroll across each of the bridges at least once, and return to her hotel on the South Bank at the end of the trip. What is the cheapest possible cost of such a trip?
    a) $10
    b) $0
    c) $11
    d) $12
    e) $9
    f) None of the above

Solve the problem.

40. The basic rule in Fleury's algorithm is
    a) never travel across a bridge of the original graph.
    b) never travel across a bridge of the untraveled part of the graph.
    c) always travel across a bridge of the original graph if you can.
    d) always travel across a bridge of the untraveled part of the graph if you can.
    e) travel across a bridge of the untraveled part of the graph only if you have to.
    f) travel across a bridge of the original graph only if you have to.

Suppose you have the following project digraph. (The numbers in parenthesis represent hours.)



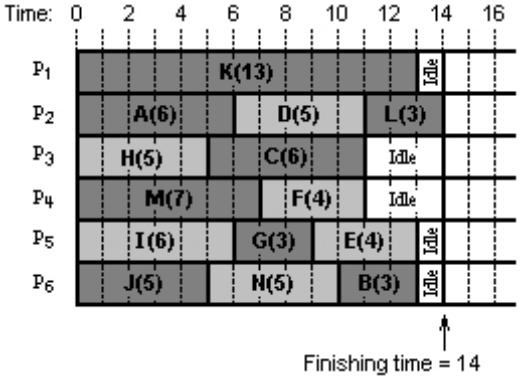
41. The length of the critical path of this project digraph is
    a) 16 hours.
    b) 11 hours.
    c) 14 hours.
    d) 12 hours.
    e) None of the above

42. Write the critical-time priority list for the above digraph.

43. The optimal completion time for this project using seven processors is
    a) 16 hours.
    b) 12 hours.
    c) 14 hours.
    d) 11 hours.
    e) None of the above

Solve the problem.

44. Which of the following is a priority list that would produce the schedule of independent tasks shown below?



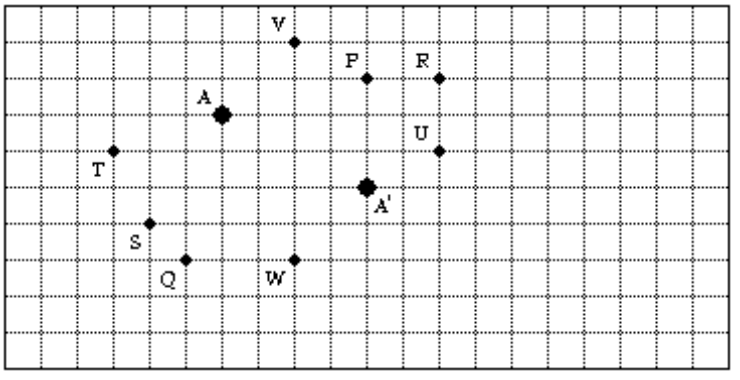
    a) K, A, H, M, I, J, C, N, D, G, B, F, E, L
    b) K, A, H, M, I, J, D, C, F, G, N, L, E, B
    c) A, B, C, D, E, F, G, H, I, J, K, L, M, N
    d) K, A, D, L, H, C, M, F, I, G, E, J, N, B
    e) All of the above, because the tasks are independent.
    f) None of the above

45. Consider a project consisting of five tasks (A through E) with the following processing times (in hours): A(10), B(10), C(5), D(5), and E(2). If a schedule with four processors has a completion time of 20 hours, what is the total idle time in the schedule?
    a) 9 hours.
    b) 12 hours.
    c) 80 hours.
    d) 48 hours.
    e) 32 hours.
    f) 10 hours

46. Recall: A digraph is called *symmetric* if whenever there is an arc from vertex X to vertex Y, there is also an arc from Y to X. A digraph is called *totally asymmetric* if whenever there is an arc from X to Y, there is no arc from Y to X. To the left of each of the following, write whether the digraph is Symmetric (write S), Totally asymmetric (write T), or Neither (write N).
    a) A digraph in which the vertices represent the students in a classroom, where there is an arc from vertex X to vertex Y if X knows the age of Y.
    b) A digraph in which the vertices represent the students in a classroom, where there is an arc from vertex X to vertex Y if X is older than Y.
    c) A digraph in which the vertices represent the students in a classroom, where there is an arc from vertex X to vertex Y if X has the same first name as Y.

Solve the problem.

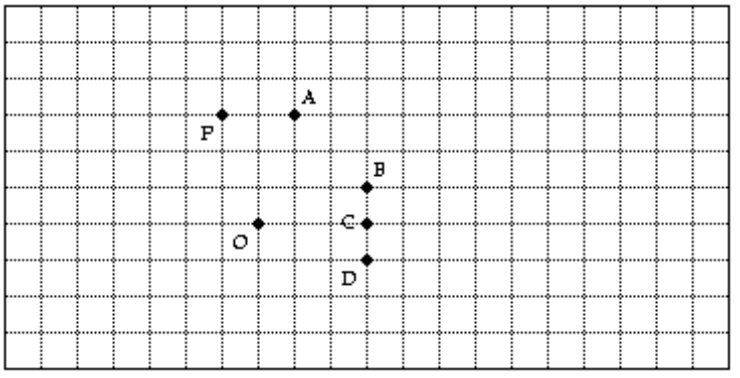
47. In the figure below, point A' is the image of point A under a glide reflection whose axis of reflection passes through point Q. The vector of translation for this glide reflection is



    a) one unit to the right, one unit up.
    b) four units to the left, two units up.
    c) four units to the left.
    d) two units to the right, one unit down.
    e) two units up.
    f) two units down.

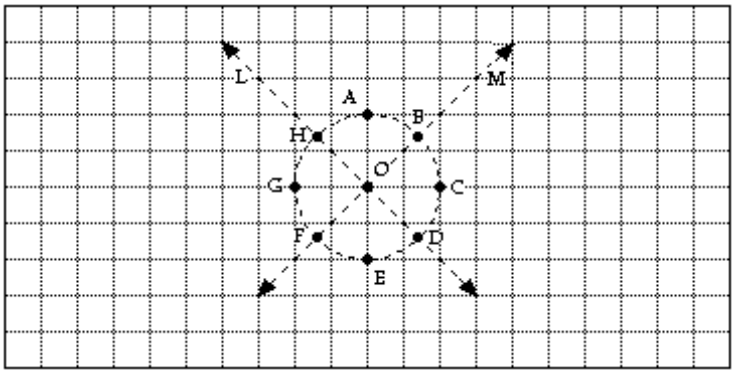
48. If an object has a 150° clockwise rotation as one of its symmetries, then it must also have as a symmetry
    a) a 60° counterclockwise rotation.
    b) a 30° counterclockwise rotation.
    c) a 30° clockwise rotation.
    d) a 90° clockwise rotation.
    e) a 60° clockwise rotation.
    f) All of the above.
    g) None of the above.

49. In the figure shown below, point A is the image of point P and point D is the image of point B under a rotation. The rotocenter of this rotation



a) is somewhere on the line OC but cannot be determined with certainty from the given information.
b) is where lines PB and AD intersect..
c) is where lines PA and BD intersect.
d) is the point O.
e) is the midpoint of the line connecting the midpoints of PA and BD.

50. Which rigid motions have at least two fixed points?
a) All rotations and reflections.
b) All rigid motions except rotations.
c) All proper rigid motions.
d) All improper rigid motions.
e) All translations and reflections.
f) All reflections .



51. In the above figure, a reflection about the line L followed by a 90° clockwise rotation having rotocenter O gives
a) a 90° counterclockwise rotation having rotocenter O .
b) a 180° clockwise rotation having rotocenter O.
c) a 45° clockwise rotation having rotocenter O.
d) a reflection in line AE.
e) a reflection in line GC.
f) a reflection in line M.