Mathematics 105: Math as a Liberal Art. Final Name_____
Instructor: Ramin Naimi Spring 2008
Close book. Closed notes. NO CALCULATORS.

Solve the problem.

1. Which of the following four graphs is a tree?

Graph 1 Graph 2 Graph 3 Graph 4

a) Graph 1 and Graph 3
b) Graph 1 and Graph 4
c) Graph 2 and Graph 3
d) Graph 2 and Graph 4
e) Graph 3 and Graph 4
f) None of the above

2. A tree is
   a) any graph with that is connected and has no bridges.
   b) any graph that is connected and has no circuits.
   c) any graph that has no circuits.
   d) any graph that is not connected and has no bridges.
   e) any graph that has at least one bridge.
   f) None of the above

3. A tree is
   a) any graph with one component and no circuits.
   b) any connected graph whose edges number one fewer than its vertices.
   c) any graph for which there is one and only one path joining any two vertices.
   d) All of the above define a tree.
   e) None of the above

4. The number of edges in a tree with 49 vertices is
   a) 2^{49}.
   b) 49.
   c) 50.
   d) cannot be determined without further information.
   e) 48.
   f) A tree cannot have 49 vertices because every tree has an even number of vertices.

5. Assume that G is a graph with no loops or multiple edges. For each of the following, choose the option that best applies: (I) G is definitely a tree; (II) G is definitely not a tree; (III) G may or may not be a tree.
   a) G has 14 vertices, 13 edges, and no circuits.
   b) G has 14 vertices, and every edge in G is a bridge.
   c) G has 14 vertices, and they all have degree at least 2.
   d) G has 14 vertices and is connected.
   e) G has 14 vertices, and the degree of each vertex is either 1 or 2..
   f) G has 14 vertices; one vertex has degree 13 and the other vertices each have degree 1.

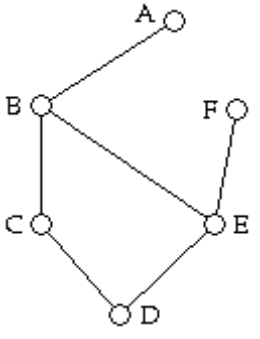
6. Suppose T is a tree with 41 vertices. Then
    a) T has no bridges.
    b) T has one bridge.
    c) T can have any number of bridges.
    d) T has 40 bridges.
    e) None of the above

7. Suppose G is a graph with 39 vertices and 38 edges. Then
    a) G must be a tree.
    b) G cannot have any circuits.
    c) G is either a tree or it is not connected.
    d) G cannot have more than one path joining any two vertices.
    e) None of the above

8. A graph has 37 vertices and has the property that there is one and only one path joining any two vertices of the graph. Which of the statements [A), B), C), or D)] is not true?
    a) Every edge of the graph must be a bridge.
    b) G must have 36 edges.
    c) G must be connected.
    d) G cannot have any circuits.
    e) All of the above statements are true.
    f) Only two of the above statements are true.

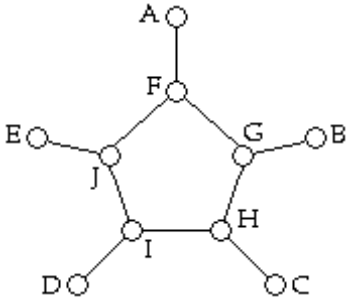
9. Which of the graphs described below must be a tree?
    a) An ancestry graph: People are the vertices; birthparents are joined to their offspring by edges.
    b) A minimal spanning subgraph of a weighted network representing cities and distances between them.
    c) The students at Tasmania State University are the vertices; edges represent whether two students have had a class together or not.
    d) All of the above graphs must be trees.
    e) Exactly two of the above graphs must be trees.
    f) None of the above graphs must be trees.

10. Every spanning tree of the following graph contains edge



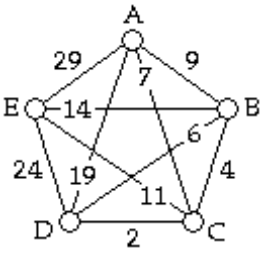
    a) DE
    b) BC
    c) CD
    d) BE
    e) None of the above

11. How many spanning trees does the following graph have?



a) 4
b) 6
c) 5
d) 7
e) None of the above

The question(s) that follow refer to the problem of finding the minimum spanning tree for the weighted graph shown below.



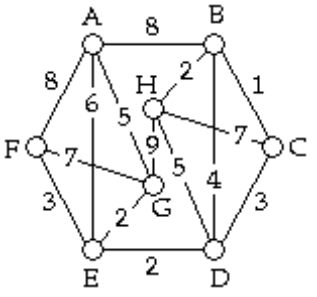
12. Using Kruskal's algorithm, which edge should we choose second?
a) BC
b) BD
c) AC
d) CD
e) AB
f) None of the above

13. Using Kruskal's algorithm, which edge should we choose third?
a) BC
b) BD
c) AC
d) CD
e) AB
f) None of the above

14. Which of the following edges of the given graph are not part of the minimum spanning tree?
a) CD
b) AC
c) BD
d) BC
e) EC
f) None of the above

15. The total weight of the minimum spanning tree is
    a) 24.
    b) 68.
    c) 20.
    d) 26.
    e) 125.
    f) 13

New telephone lines must be installed to connect 8 cities (A, B, C, D, E, F, G, and H). Due to government regulations, the only possible connections that are allowed are shown by the edges on the graph below. The numbers on the edges indicate the cost (in millions of dollars) of each possible connection. The telephone company wishes to connect the cities in the cheapest possible way.



16. Using Kruskal's algorithm, which edge is chosen last?
    a) CD or EF
    b) BC
    c) HG
    d) AG
    e) BH, EG or DE
    f) AB or AF

17. Using Kruskal's algorithm, which edge is chosen second to last?
    a) BH, EG or DE
    b) CD or EF
    c) AF
    d) BC
    e) AB or AF
    f) CH or FG

18. The number of bridges in the optimal solution to this problem is
    a) 8.
    b) 9.
    c) 7.
    d) 0.
    e) None of the above

Use the mileage chart shown below to find the minimum spanning tree for the 5 cities of Boston, Buffalo, Chicago, Columbus, and Louisville.

	Boston	Buffalo	Chicago	Columbus	Louisville
Boston	*	446	963	735	941
Buffalo	446	*	522	326	532
Chicago	963	522	*	308	292
Columbus	735	326	308	*	209
Louisville	941	532	292	209	*

19. Using Kruskal's algorithm which edge is chosen third?
    a) Buffalo - Columbus.
    b) Boston - Chicago.
    c) Boston - Columbus.
    d) Columbus - Louisville.
    e) Columbus - Chicago

20. Which of the following edges is not in the minimum spanning tree?
    a) Columbus - Chicago.
    b) Columbus - Louisville.
    c) Buffalo - Columbus.
    d) Boston - Buffalo.
    e) All of the above are in the minimum spanning tree.

Solve the problem.

21. Which of the following statements is true about Kruskal's algorithm.
    a) It is an inefficient algorithm, and it never gives the minimum spanning tree.
    b) It is an efficient algorithm, and it always gives the minimum spanning tree.
    c) It is an efficient algorithm, but it doesn't always give the minimum spanning tree.
    d) It is an inefficient algorithm, but it always gives the minimum spanning tree.
    e) None of the above

22. Suppose a graph G has V vertices and E edges, such that $V = E + 1$. If G is definitely a tree or definitely not a tree, explain why; if it can be either, draw examples of both.