Closed book. Closed Notes. 20 points per problem. Please write very legibly.

All of the following problems concern Statement Calculus (not Predicate Calculus).

1. Is each of the following True or False? Prove your answers.

    (a) Let Γ be a (possibly infinite) set of formulas. If Γ is not satisfiable, then it contains a formula A that's not satisfiable.

    (b) Let A be a formula and Γ and Δ (possibly infinite) sets of formulas. If $\Gamma \vdash A$ and, for every formula $B \in \Gamma$, $\Delta \vdash B$, then $\Delta \vdash A$.

2. (a) State the Strong Completeness and the Strong Soundness Theorems. Then use these theorems to prove the following:

Let B be a formula, and Γ a set of formulas. If $\Gamma \models B$, then for some finite subset Δ of Γ, $\Delta \models B$.

(b) State the Compactness Theorem. Then use the Compactness Theorem (but not the Strong Completeness and the Strong Soundness Theorems) to prove the following:

Let B be a formula, and Γ a set of formulas. If $\Gamma \models B$, then for some finite subset Δ of Γ, $\Delta \models B$.