**Quiz 5.** Math 350, Mathematical Logic. Name: ______________________

Instructor: Ramin Naimi Mon 5 Mar 2004

Closed book. Closed Notes. 20 points per problem. Please write very legibly.

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Do **only two** of the following problems.

1. (The notation and terminology in this problem are according to the homework problems on the FM Game.)

   True or False: Any wff that can be obtained using FM1-4 (listed below) is always even. Prove your answer.

FM1: For any wff $A$, you may write $f(A)A$.

FM2: If you've already written a wff $A$, then you may write BA for any wff B.

FM3: If you've already written a wff of the form $AA$, then you may write $A$.

FM4: If you've already written two wff's of the form $AB$ and $f(A)C$, then you may write $BC$.

2. State and prove the Deduction Theorem.

3. Complete the statement and proof of Lemma 9.2.1 by providing the missing details below wherever you see "**???**". The missing details may consist of one or many sentences. The notation used here is what we used in class, which is slightly different from the book's. If you prefer, you may use the book's notation, in which case you'll have to rewrite in the book's notation what's written here, rather than just provide the missing details. You may assume and use $B \vdash \neg\neg B$.

   Lemma 9.2.1: Let $A$ be any formula, and $P_1, \cdots, P_n$ denote the statement variables in $A$. Suppose we construct a truth table for $P_1, \cdots, P_n, A$. For each row $i$, let $P_{ij} = \begin{cases} P_j & \text{if } P_j \text{ is } \texttt{T} \text{ in row } i \\ \neg P_j & \text{otherwise} \end{cases}$, and let $A_i = \begin{cases} A & \text{if } A \text{ is } \texttt{T} \text{ in row } i \\ \neg A & \text{otherwise} \end{cases}$. Then for every $i$, **(a)???**.

   Proof: By induction on the length of $A$, where *length* is defined as **(b)???**.

   *Base Step.* **(c)???**

   *Induction Step.* Suppose $A$ has length $L$, and assume the lemma is true for all formulas with **(d)???**.

   There are two cases: Case 1. $A$ is of the form $\neg B$. Case 2. $A$ is of the form **(e)???**.

   Case 1. **(f)???**.

   Case 2. Omit (do not prove this case).

4. State and prove the Completeness (Adequacy) Theorem. In your proof you may assume and use the Deduction Theorem, Lemma 9.2.1 (in the previous problem), and the following: $\vdash (B \to A) \to ((\neg B \to A) \to A)$.