Quiz 3. Math 350, Mathematical Logic. Name: ____________________
Instructor: Ramin Naimi Fri 27 Feb 2004

Closed book. Closed Notes. Please write very legibly.

Extra Credit problems do not carry any points; so do not spend any time on them unless you're sure you've done your best with the problems that do carry points.

1. Using the letters indicated for predicates, and whatever symbols of arithmetic (for example, "+" and "<") may be needed, translate the following.

   (a) (10 points) No prime number greater than 2 is divisible by an even number. (Px: x is prime; Ex: x is even; Dxy: x divides y.)

   (b) (10 points) Only judges admire judges. (Jx: x is a judge; Axy: x admires y.)

2. (20 points) Using the letters indicated for predicates, and whatever symbols of arithmetic (for example, ">" and "≤") may be needed, translate each of the following; then use tautologies 1-40 to show the two statements are equivalent.

   (i) There is no positive real number that is less than or equal to every positive real number. (Rx: x is a real number.)

   (ii) Every positive real number is greater than some positive real number.

3. (20 points) In the following, let x and y be distinct variables, $A(x,y)$ and $B(x)$ be any formulas, and A be any formula not containing any free occurences of x.

   Determine whether each of the following is a valid statement. Prove your answers. (You may use tautologies 1-40.)

   (a) $(\forall y)(\exists x)A(x,y) \to (\exists x)(\forall y)A(x,y)$

   (b) $\forall x(A \to B(x)) \leftrightarrow A \to \forall x B(x)$

4. Extra Credit Problem (0 points):

   In the following, let x be a variable, $B(x)$ be any formula, and A be any formula not containing any free occurences of x.

   Determine whether each of the following is a valid statement. Prove your answers.

   You may use tautologies 1-40. You may also find the following useful:

   Theorem 8.1. Let $A(x)$ be a formula which is free for y. Then:
   (I) $\models (x)A(x) \to A(y)$.
   (II) $\models A(y) \to (\exists x)A(x)$.

   (a) $\forall x(B(x) \to A) \to (\forall x B(x) \to A)$

   (b) $(\forall x B(x) \to A) \to \forall x(B(x) \to A)$