

Closed book. Closed Notes. Please write very legibly.

1. (20 points) Determine the validity or invalidity, whichever might be the case, of the following argument. If it is valid, give a formal proof (using rules p and t only). If it's invalid, find a truth assignment that demonstrates this.

Either Sally and Bob are the same age (S) or Sally is older than Bob (O). If Sally and Bob are the same age, then Nancy and Bob are not the same age ($\neg N$). If Sally is older than Bob, then Bob is older than Walter (W). Therefore, either Nancy and Bob are not the same age or Bob is older than Walter.

2. (a) (5 points) Give the definition of what it means for a set of formulas $\{A_1, \dots, A_n\}$ to be satisfiable.
- (b) (10 points) Let A_1 denote the formula $P \rightarrow (Q \rightarrow R)$, and A_2 the formula $(P \rightarrow Q) \rightarrow R$. Is the set $\{A_1, \neg A_2\}$ satisfiable? If so, give a truth assignment to prove it; otherwise give an indirect proof (proof by contradiction, or conditional proof) to show it's not satisfiable.

3. (5 points) Do only one of (a) or (b), but *not both*. Please circle the one you're choosing.

(a) Give a formal proof (using rules p and t) for

$$A \rightarrow B, \neg A \rightarrow B \models B$$

(b) Give a formal proof (using rules p and t) for

$$\neg C \rightarrow A \vee \neg B \models \neg A \wedge B \rightarrow C$$