Quiz 1. Math 350, Mathematical Logic.   Name: _____

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Closed book. Closed Notes. 20 points per problem. Please write very legibly.

Extra Credit problems do not carry any points; so do not spend any time on them unless you're sure you've done your best with the problems that do carry points.

1. Let the formula
$$L \vee M \rightarrow (S \leftrightarrow G \wedge \neg R)$$
denote the statement

> If either labor or management is stubborn, then the strike will be settled iff the government obtains an injunction, but troops are sent into the mills.

Determine whether the above statement is true or not, if it is known that

> If the government obtains an injunction, then troops will be sent into the mills. If troops are sent into the mills, then the strike will not be settled. The strike will be settled. Management is stubborn.

Explain your work.

2. (a) Write a formula A in terms of P, Q, and R that has the following truth table. (Explanation and justification not necessary.)

P	Q	R	A
T	T	T	T
T	T	F	F
T	F	T	F
T	F	F	T
F	T	T	F
F	T	F	F
F	F	T	T
F	F	F	F

(b) Write a formula B in terms of P, Q, and R and using only the connectives $\neg$ and $\wedge$, such that B eq $(P \rightarrow Q \vee R)$. Briefly explain and justify your work (but full rigor is not necessary).

3. Use tautologies 20 and 32 given below (with Theorem 3.1 and/or Corollary 3.2, whenever necessary) to prove that
$$\models (A \leftrightarrow B) \leftrightarrow (\neg A \leftrightarrow \neg B).$$

20. $\models (A \rightarrow B) \leftrightarrow (\neg B \rightarrow \neg A)$.

32. $\models (A \leftrightarrow B) \leftrightarrow (A \rightarrow B) \wedge (B \rightarrow A)$.

Justify every step.

4. Extra Credit Problem (0 points): Show that every mapping $h : \{0,1\}^n \rightarrow \{0,1\}$ can be generated from the mapping $g : \{0,1\}^3 \rightarrow \{0,1\}$ where $g(x,y,z) = 1 + x + y + xyz \pmod 2$.

(You may find the following formulas helpful: $\neg P : 1 + P$; $P \wedge Q : P + Q + PQ$; $P \vee Q : PQ$; $P \rightarrow Q : (1 + P)Q$; $P \leftrightarrow Q : P + Q$; $P|Q : (1 + P)(1 + Q)$.)