

Closed book. Closed Notes. 20 points per problem. Please write very legibly.

Answer exactly one problem from each section. *Please circle* the five problems that you choose.

SECTION I

1. State and prove the Deduction Theorem for Statement Calculus.
 2. State and prove the Completeness (Adequacy) Theorem for Statement Calculus.
-

SECTION II

3. State the Compactness Theorem. Then use the Compactness Theorem (but not the Strong Completeness and the Strong Soundness Theorems) to prove the following in Statement Calculus:

Let B be a formula, and Γ a set of formulas. If $\Gamma \models B$, then for some finite subset Δ of Γ , $\Delta \models B$.

4. Let Γ be a set of formulas in Statement Calculus. Let $\Delta = \{\neg A \mid A \in \Gamma\}$. Prove or disprove each of the following:
 - (a) If Δ is inconsistent, then Γ is consistent.
 - (b) If Γ is not satisfiable, then it contains an inconsistent finite subset.
-

SECTION III

5. Prove the Soundness Theorem for Statement Calculus.
 6.
 - (a) Prove that Statement Calculus is consistent.
 - (b) Prove that Predicate Calculus is consistent.
-

SECTION IV

7. Use the axioms and rules of inference of Predicate Calculus (PC1-7, MP, Generalization) plus Peano's axioms (in the handout) to prove $0 + 0' = 0'$. Do not use any derived rules of inference.
8. Translate the following into a formula. Then prove or disprove it. You may use Tautologies 1-40 and rules *ug*, *us*, *eg*, *es*.

No Republican or Democrat is a Socialist. There is a Socialist. Therefore, not everyone is a Republican. (Rx : x is a Republican; Dx : x is a Democrat; Sx : x is a Socialist.)

SECTION V

9. Is $\forall x(A \rightarrow B) \rightarrow (\forall xA \rightarrow \forall xB)$ a tautology (a valid formula) in Predicate Calculus? Either explain why it is true in every interpretation, or give an interpretation that makes it false.
10. Is $(\forall xA \rightarrow \forall xB) \rightarrow \forall x(A \rightarrow B)$ a tautology (a valid formula) in Predicate Calculus? Either explain why it is true in every interpretation, or give an interpretation that makes it false.