

1. Come up with at least one problem involving one or more of: consistency, satisfiability, tautology, contradiction, formal theorem,  $\vdash$ ,  $\models$ , Completeness Theorem, Soundness Theorem, Compactness Theorem, other related topics and theorems. Your problem should not be directly equivalent to any of the problems in HW 19 or 20; in fact, the more original and different it is, the better. If you can, also write up a solution to your problem.
2. Give an informal but clear and detailed explanation (through an example) of how PC5 (page 389) could be false if there were a free occurrence of  $x$  in  $A$  in the scope of a  $\forall y$ .
3. (a) Recall that if  $x$  is free in  $A$ , then  $A$  is defined to be True in an interpretation iff  $(\forall x)A$  is True in that interpretation.  
Do there exist formulas  $A$  and  $B$  and an interpretation such that  $A \rightarrow B$  is True but  $\forall x A \rightarrow B$  is not True, where  $x$  is free in  $A$ ? (Be careful:  $\forall x A \rightarrow B$  is not the same as  $\forall x (A \rightarrow B)$ .)  
(b) Give an informal but clear and detailed explanation (through an example) of how PC4 (page 389) could be false if there were a free occurrence of  $x$  in  $A$ .
4. Prove the Soundness Theorem for Predicate Calculus. (The book's proof of Theorem 9.3.3 leaves out a lot of details. Either provide detailed explanations for it, or write up a proof entirely in your own style and wording.)