1. Use Theorem 9.2.7 on page 385 (which says every consistent set of formulas is satisfiable) to prove the Compactness Theorem.

2. (Extra Credit) Complete the following "direct" proof of the Compactness Theorem.

   Compactness Theorem: Let Γ be a set of formulas. If every finite subset of Γ is satisfiable, then Γ is satisfiable.

   Proof: If Γ is finite, then there is nothing to prove. So assume $\Gamma = \{A_1, A_2, A_3, \cdots\}$ is infinite. (We are implicitly asserting that Γ is countable, which is true since there are only countably many symbols and statement variables and every formula is a finite string of symbols and statement variables.) By hypothesis, for each $i = 1, 2, \cdots$, there exists a truth assignment ϕ_i that makes each of $A_1, \cdots, A_i$ true. We will construct a truth assignment ϕ that makes A_j true for all j. The construction is recursive:

   Base Step. Let $P_1, P_2, \cdots$ denote all the statement variables. Then there are either infinitely many ϕ_i that make P_1 true, or infinitely many ϕ_i that make P_1 false (or possible both) **(Why?)**. In the former case, let $\phi(P_1) = \mathtt{T}$; otherwise let $\phi(P_1) = \mathtt{F}$.

   Let $S_1 = \{\phi_j \mid \phi_j(P_1) = \phi(P_1)\}$. Then S_1 is infinite **(Why?)**.

   Recursion Step. Suppose we have defined $\phi(P_1), \cdots, \phi(P_k)$. Suppose also that the set $S_k = \{\phi_j \mid \phi_j(P_k) = \phi(P_k)\}$ is infinite. Then we define $\phi(P_{k+1})$ as follows and show that S_{k+1} is infinite.

   $$\phi_{k+1} = \begin{cases} \mathtt{T} & \text{if there are infinitely many } \phi_j \text{ such that } \phi_j(P_{k+1}) = \mathtt{T} \\ \mathtt{F} & \text{otherwise} \end{cases}$$

   Prove that S_{k+1} is infinite.

   Prove that ϕ satisfies Γ.

   Answer all the **(Why?)**'s above.