Midterm #3. Math 350, Logic.
Instructor: Ramin Naimi

Name: _____
Wed 18 Apr 2001

Do **only four** of the following five problems. Closed book. Closed Notes. Only the Definitions-Theorems handout allowed. 25 points per problem. Please write very legibly.

1. (a) Let A be a formula whose only connective is $\vee$. Can A be a tautology? Justify your answer.
   (b) Let A be a formula whose only connective is $\leftrightarrow$. Can A be a tautology? Justify your answer.
2. Find a model for the sentence $\forall x[R(c,x) \rightarrow R(F(c),F(x))]$, where R is a relation symbol, F a function symbol, and c a constant symbol. Briefly explain your work.
3. Show $[\forall x A \rightarrow \forall x B] \rightarrow [\forall x(A \rightarrow B)]$ is not logically valid.
4. Let Γ be a set of formulas, and A and B arbitrary formulas in Propositional Logic. Is the following true or false? If $\Gamma \cup \{B\} \models A$ and $\Gamma \cup \{B\} \models \neg A$, then $\Gamma \models \neg B$. Prove your answer.
5. Let Γ be a set of formulas, and A and B arbitrary formulas in Propositional Logic. Is the following true or false? If Γ is complete and $\Gamma \vdash (A \rightarrow B)$, then $\Gamma \vdash \neg A$ or $\Gamma \vdash B$. (To say Γ is complete means for every A, Γ proves A or its negation.) Prove your answer.

Axioms and Rules of Inference for Propositional Logic
(You may not need the following for any of the problems above.)

Axioms: $(\neg A \vee A)$.

Rules of Inference:

Associative Rule: $\frac{(A \vee (B \vee C))}{((A \vee B) \vee C)}$ Contraction Rule: $\frac{(A \vee A)}{A}$

Expansion Rule: $\frac{A}{(B \vee A)}$ Cut Rule: $\frac{(A \vee B), (\neg A \vee C)}{(B \vee C)}$

Some Derived Rules of Inference:

New Associative Rule: $\frac{((A \vee B) \vee C)}{(A \vee (B \vee C))}$ Commutative Rule: $\frac{(A \vee B)}{(B \vee A)}$

New Expansion Rule: $\frac{A}{(A \vee B)}$ Modus Ponens: $\frac{A, (A \rightarrow B)}{B}$

(You may also use other derived rules of inference, if you recall them accurately.)

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Please circle the four problems you are choosing. **1**. **2**. **3**. **4**. **5**.