Midterm #1. Math 350, Logic. Name: ____________________
Instructor: Ramin Naimi Fri 16 Feb 2001

Closed book. Closed Notes. 25 points per problem. Please write very legibly.

1. (25 points) Show $\vdash (A \to (A \vee B))$.

   Axioms: $(\neg A \vee A)$.

   Rules of Inference:

   Associative Rule: $\frac{(A\vee(B\vee C))}{((A\vee B)\vee C)}$ Contraction Rule: $\frac{(A\vee A)}{A}$

   Expansion Rule: $\frac{A}{(B\vee A)}$ Cut Rule: $\frac{(A\vee B),\,(\neg A\vee C)}{(B\vee C)}$

   Some Derived Rules of Inference:

   New Associative Rule: $\frac{((A\vee B)\vee C)}{(A\vee(B\vee C))}$ Commutative Rule: $\frac{(A\vee B)}{(B\vee A)}$

   New Expansion Rule: $\frac{A}{(A\vee B)}$ Modus Ponens: $\frac{A,\,(A\to B)}{B}$

   You may (but do not need to) also use other derived rules of inference, if you recall them accurately.

2. (a) (5 points) Give the definition of **tautological consequence**. Write a complete and grammatically correct sentence.

   (b) (20 points) True or False: If A and B are formulas such that $\{A\} \models B$, then $\{\neg A\} \models \neg B$. Prove your answer.

3. (a) (5 points) Let Γ be a set of formulas, A a formula. Give the definition of $\Gamma \vdash A$. Write a complete and grammatically correct sentence.

   (b) (20 points) Show if $\Gamma \vdash (A \to B)$, then $\Gamma \cup \{A\} \vdash B$. Hint: Use one or more of the (derived) rules of inference.

4. (a) (5 points) Give the definition of what it means for a set of connectives to be **adequate**. Write a complete and grammatically correct sentence.

   (b) (15 points) The connective NOR, $\downarrow$, can be defined as: $(p \downarrow q) \cong (\neg p \wedge \neg q)$. Write the formula $(p \wedge q)$ in terms of $\downarrow$. Explain your work.

   (c) (5 points) Describe an algorithm (a systematic method) for the following: given an arbitrary n-ary truth function $G : \{\mathrm{T}, \mathrm{F}\}^n \to \{\mathrm{T}, \mathrm{F}\}$, find a formula A in terms of $\downarrow$ only, such that $H_A = G$.