Final Exam. Math 350, Logic.
Instructor: Ramin Naimi

Name: ______________________
Wed 9 May 2001, 1:00-4:00 PM.

Do **only six** of the following problems. 20 points per problem. Closed book. Closed Notes. Only the Definitions-Theorems handout allowed. No electronic devices other than watches allowed. Please write very legibly.

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1. (a) Write the following in L_{NN}: The only prime number that's even is 2.
   (b) Write the following in L_{ST}: Every nonempty set has at least two distinct subsets.
2. Let Γ and Δ be sets of formulas in a formal system F, and A an arbitrary formula in F. Prove that if Γ proves every formula of Δ, and $\Delta \vdash A$, then $\Gamma \vdash A$.
3. Let Γ be a finite nonempty set of formulas, and A an arbitrary formula in Propositional Logic. Is "$\Gamma \vdash A$" decidable? If not, is it semidecidable? Prove your answer.
4. Is the formula $[\forall x A \leftrightarrow \forall x B] \rightarrow [\forall x(A \leftrightarrow B)]$ logically valid? Prove your answer.
5. Let Γ and Δ be sets of formulas in a first order language L. One of the following is true, the other false. State which one is true and prove it. (You don't need to say anything about the one that's false.)
   (a) If every model of Γ is a model of Δ, then every theorem of Δ is a theorem of Γ.
   (b) If every model of Γ is a model of Δ, then every theorem of Γ is a theorem of Δ.
6. Let A, B, and C be formulas in a first order language L.
   True or false? If each of the sets $\{A, B\}$, $\{B, C\}$, and $\{C, A\}$ is consistent, then $\{A, B, C\}$ is consistent. Prove your answer.
7. Let Γ and Δ be sets of formulas in a first order language L, and B an arbitrary formula in L.
   (a) True or false? If $\Gamma \cup \Delta \vdash B$ then $\Gamma \vdash B$ or $\Delta \vdash B$. Prove your answer.
   (b) Recall that we say Γ is *consistent* if there is no formula A such that $\Gamma \vdash A$ and $\Gamma \vdash \neg A$. And we say B is *consistent with* Γ if $\Gamma \cup \{B\}$ has a model.
   True or false? B is consistent with Γ iff $\Gamma \cup \{B\}$ is consistent. Prove your answer.
8. Use the Peano Postulates listed below to prove each of the following.
   (a) $1 + 1 = 2$.
   (b) For every $a \in \mathbb{N}$, $1 + a = S(a)$.

   Peano Postulates

   The following hold for all $a, b \in \mathbb{N}$: P1: $S(a) \neq 0$. P2: $S(a) = S(b) \rightarrow a = b$ P3: Let $A \subset \mathbb{N}$ satisfy (1) $0 \in A$; and (2) if $a \in A$ then $S(a) \in A$. Then $A = \mathbb{N}$. P4: $a + 0 = a$. P5: $a + S(b) = S(a + b)$. P6: $a \times 0 = 0$. P7: $A \times S(b) = (a \times b) + a$.
9. Is the collection of all infinite subsets of $\mathbb{N}$ countable or uncountable? Prove your answer.
