Decidable problems

Example 1. For $n \in \mathbb{N}$, let $R(n)$ say: "n is the sum of two consecutive squares."

Q: Is $R(13)$ true or false? Ans: True, b/c $13 = 2^2 + 3^2$.

Q: Is $R(1000)$ true or false?

Q: Give an algorithm for determining whether $R(n)$ is true or false for any given n. No calculators, computers, etc. Paper and pencil only!

Definition 1. (Informal) A relation is said to be **decidable** if there is an algorithm for deciding whether the relation is true or false for any given input; i.e., given any input, the algorithm will stop after finitely many steps and give an output of YES or NO (or T or F).

Example 2. (Example 1 from book)

Q: What's a palindrome? Ans: A word that's "left-right" symmetric. E.g.: Palindrome: abba, cac. Not palindrome: abcd, abab.

Q: Give an algorithm for deciding whether a word is a palindrome. Ans: See book.

Semidecidable problems

Definition 2. (Informal) A relation R is **semidecidable** if there is an algorithm that, given any input x, stops with output YES if $R(x) = \mathrm{T}$, and doesn't stop if $R(x) = \mathrm{F}$.

Note. For an n-ary relation we would have n inputs: $R(x_1, \cdots, x_n)$.

Example 3. Recall the formal system MIU:

Axiom: MI.

Rules of Inference: R1: $xI \to xIU$. R2: $Mx \to Mxx$. R3: $xIIIy \to xUy$. R4: $xUUy \to xy$.

Q: Let $R(x) =$ "x is a theorem of MIU." Show R is semidecidable.

Q: Is R decidable?

Remark. There is nothing special about MIU in the above example: in every formal system F, the relation $R(A) =$ "A is a theorem of F" is semidecidable.

HW #22, due Mon 30 Apr: Read: Section 1.7: p. 30-32. Do: p. 38: 1, 2, 9.