Recall the following table from a few weeks ago:

Propositional Logic:	Semantics: (Chapter 2) Truth values	Syntax: (Chapter 3) Formal systems, Proofs
First-Order Logic (FOL):	Semantics: (Chapter 4) Truth values Models, Interpretation	Syntax: (Chapter 5) Formal Systems, Proofs

A priori, "true" and "provable" are unrelated: one does not necessarily have to imply the other. We proved in Chapter 3, though, that they do imply each other; what were the names of these results? Soundness and Adequacy – for P.

In Chapter 5, instead of Propositional Logic, we work with: First Order Logic = First Order Language + axioms and rules of inference.

Abbreviations: First Order Logic = $\mathrm{FO_L}$ (or sometimes just FO). First Order Language: FOL

Let's try to guess what the Soundness and Adequacy Theorems will say now...

Main results of Chapter 5

1. $\vdash A$ iff A is logically valid.
2. $\Gamma \vdash A$ iff A is true in every model of Γ.
3. Γ is consistent iff it has a model.

$\mathrm{FO_L}$ has the same axioms and rules of inference as P, plus a few more, for working with quantifiers.

Axioms of $\mathrm{FO_L}$

Propositional Axioms: $\neg A \vee A$.

Substitution Axioms: $\forall x A \to A_x[t]$. (Special case: $\forall x A \to A$.)

Reflexive Axioms: $\forall x(x = x)$.

Equality Axioms $(x = t) \to (A \leftrightarrow A_x[t])$.

(Wherever $A_x[t]$ appears, it is subject to the requirement that t be substitutable for x in A.)

Rules of inference of $\mathrm{FO_L}$

Same rules of inference as for P: ASSOC, EXP, CUT, CTN; plus:

Add $\forall$ Rule: $\dfrac{A \vee B}{A \vee \forall x B}$ (x not free in A).

Note. Our book contains several derived rules of inference and theorems of $\mathrm{FO_L}$ that are useful in writing proofs in $\mathrm{FO_L}$. You do not need to memorize any of them. But you should refer to them for doing HW problems, and be able to use them on the exam (they will be provided).

Example 1. [Page 177, problem 1(1)] Show: $\vdash \forall x A \to \exists x A$.

Proof:

1. $\forall x A \to A$ SUBST AX
2. $\neg\forall x A \vee A$ DEF $\to$
3. $A \to \exists x A$ SUBST THM
4. $\neg A \vee \exists x A$ DEF $\to$
5. $A \vee \neg\forall x A$ CM(2)
6. $\neg\forall x A \vee \exists x A$ CUT(4,5)
7. $\forall x A \to \exists x A$ DEF $\to$

Read page 173 (Tautologies of $\mathrm{FO_L}$) before doing the HW (problem 8).

HW #19, due Mon 16 Apr: Read sec 5.1. Do: p. 177: 1(2,3), 6, 8, 12.