Summary: Given any FOL (such as Lnn, Lst, etc.), we can interpret the symbols in different ways. A given formula can be true in some interpretations, and false in others. Given a set of formulas, an interpretation that makes every formula in that set true is called a model of that set.

Example 1. Let L be the following language: no constants; no functions; one binary relation, R. Let A be the formula $\forall x[\neg(xRx) \wedge \exists y(yRx)]$.

Q: Is A true or false in the following **interpretation** I? **Domain** (or *universe*): $D_I = \{0, -1, -2, -3, \cdots\}$ (nonpositive integers); R_I is interpreted as $>$.

Ans: No; the first half of A (before $\wedge$) is satisfied, but the second half isn't: for $x = 0$ there is no y such that $y > x$ (in the given domain).

Q: Find an interpretation that makes A true. Ans: There are many possibilities; for the same domain as above, let R_I be $<$. Or, change the domain to $D_I = \mathbb{N}$, and let R_I be $>$.

- An interpretation that makes A true is called a **model** of A.

Q: Can you find a **finite model** for A, i.e., an interpretation with a finite domain?

Ans: Again, lots of possibilities; here's one: Let $D_I = \{1, 2, 3\}$; let R_I be defined by: $1R_I2, 2R_I3, 3R_I1$ (or more precisely, $R_I = \{(1, 2), (2, 3), (3, 1)\}$).

Example 2. Let L be the following language: two constants, c, d; one unary function, F; one binary relation, R.

Q: Find a model for the set $\Gamma = \{A, B, C\}$, where A is the formula $F(c) = d$, B is the formula $\forall x \exists y(xRy)$, C is the formula $\forall x \forall y \forall z[(xRy \wedge yRz) \to xRz]$.

Ans: One model (among many): Let I be the following interpretation: $D_I = \mathbb{R}$, $c_I = 0$, $d_I = 1$, $F_I(x) = x + 1$, xR_Iy iff $x \leq y$.

Now let J be the following interpretation: D_J = the set of all humans (living or dead); $F_J(x)$ = father of x; xR_Jy iff x is a sibling of y; c_J = Michael Douglas; d_J = Kirk Douglas.

Q: Are A, B, C true in this last interpretation? A is true since Kirk is Michael's father; B is true since everyone has a father; C is true since if x and y are siblings and y and z are siblings, then x and z are siblings.

Actually, J is not a model for Γ: B is not true in this interpretation! If every human had a human father, then there must have existed infinitely many humans by now!

HW # 13, due Mon 19 Mar
Read Section 4.2. Do: p. 141: 27, 28(1,2), 29, 32(2abc).