Today we'll revisit Lnn, and see a few more FOLs: Lst: FOL for Set Theory; Ls: FOL for Syllogisms; Lr: FOL for relations.

All these different FOLs share certain symbols, features, rules:

L: "Generic FOL"

1. Logical Symbols: $\neg\ \vee\ \forall\ (\ )\ ,\ =\ \ x_1\ x_2\ \cdots$.
2. Nonlogical Symbols (specific to each FOL): Constant Symbols; Function Symbols; Relation Symbols.
3. Terms: T1: Constants; variables. T2: Combined terms, using functions.
4. Formulas: F1: Combined terms or formulas using relations ("=" is a relation). F2: If A, B are formulas, so are: $\neg A$, $A \vee B$, $\forall x_i A$.

Definition 1. For any formula A, "$\exists x_i A$" stands for "$\neg\forall x_i \neg A$."

Example 1. Lnn: FOL for arithmetic. Constants: 0. Functions: S, $+$, $\times$. Relations: $<$.

Q: Is each of the following a term or formula? $x_1 + x_2$: term. $x_1 < x_2$: formula. $\forall x_1(x_1 < x_2)$: formula. $\forall x_1(x_1 + x_2)$: neither (it's an illegal expression).

Example 2. Lst: FOL for Set Theory. Constants: none. Functions: none. Relations: $\in$.

Examples: Term: x_3. Formula: $x_1 \in x_3$; $x_1 = x_3$; $\forall x_1 \in x_3$; $(x_1 \in x_3) \vee \neg(x_2 = x_1)$.

Q: We are not defining symbols such as: $\subset$, $\cap$, $\cup$, etc. Why? Ans: Can define them using $\in$ only. How?

For example, $x_1 \subset x_2$ stands for: $\forall x_3[(x_3 \in x_1) \rightarrow (x_3 \in x_2)]$.

Example 3. Write the sentence $\forall x \forall y (x \cap y) \subset y$ without using the symbols $\cap$ and $\subset$.

Ans: $\forall x \forall y \forall z (z \in x \wedge z \in y) \rightarrow z \in y$

Example 4. Russell's Paradox: Let S be the set of all sets T such that $T \notin T$.

Q: True or False: $S \in S$. Ans: Neither! Why?

Example 5. Syllogisms: Here, examples and a brief description only; read precise def in book.

All dogs are animals. Some animals can fly. Therefore some dogs can fly. (This is an invalid argument; but that's beside the point.)

Features: Always two hypotheses, one conclusion. Each sentence is of the form: Some A are B; all A are B; no A are B; some A aren't B.

Ls: FOL for Syllogisms: Constants: none. Functions: none. Relation symbols: Infinitely many, all unary may use any letter (usually uppercase).

Example for above argument: $D(x)$ = "x is a dog." $A(x)$ = "x is an animal." $F(x)$ = "x can fly."

We get: $\forall x[D(x) \rightarrow A(x)]$, $\exists x[A(x) \wedge F(x)]$, $\therefore \exists x[D(x) \wedge F(x)]$.

Example 6. Lr: FOL for relations: like Ls, except allows constant symbols. Read in book.

HW # 11, due Fri 09 Mar
Read Section 4.1. Do: p. 134: 4, 5, 6(2,4), 7.

HW # 12, due Mon 12 Mar
Read Section 4.2. Do: p. 141: 6-8, 14, 19, 21, 25(1).