Page 74, Problem 18: Prove that $\{\wedge, \vee, \rightarrow\}$ is an inadequate set of connectives.

Proof. We will show that there is no formula A using the connectives in $\{\wedge, \vee, \rightarrow\}$ only, such that $H_A = H_\neg$; this shows that $\{\wedge, \vee, \rightarrow\}$ is not adequate.

Let A be an arbitrary formula. To show that $H_A \neq H_\neg$, it's enough to show $H_A(\mathrm{T}) \neq H_\neg(\mathrm{T})$. We'll use induction on the number of symbols in A to show that $H_A(\mathrm{T}) = \mathrm{T}$, which is different from $H_\neg(\mathrm{T}) = \mathrm{F}$.

Base Step: If A has only one symbol, then it must be a propositional variable. So $H_A(\mathrm{T}) = \mathrm{T}$, as desired.

Induction Step: As induction hypothesis, assume that for any formula B with $\leq n$ symbols in it, $H_B(\mathrm{T}) = \mathrm{T}$. Suppose A has $n+1$ symbols. Then, by rule F2, A is either $(B \vee C)$ or $(B \wedge C)$ or $(B \rightarrow C)$. So each of B and C has $\leq n$ symbols, and by the induction hypothesis, $H_B(\mathrm{T}) = \mathrm{T}$ and $H_C(\mathrm{T}) = \mathrm{T}$. From the definition of $\{\wedge, \vee, \rightarrow\}$ it follows that $H_A(\mathrm{T}) = \mathrm{T}$. $\square$