Last time we saw a very simplified example of a formal system: ADD. That was preparation for studying the formal system of Propositional Logic, which is what Chapters 2 and 3 are about.

(Review from Math 140) In logic we use the following symbols: $\wedge$ for 'and'; $\vee$ for 'or'; $\neg$ for 'not'; $\rightarrow$ for 'if...then' or 'implies', and $\leftrightarrow$ for 'iff'.

These five symbols are called **logical connectives**.

Example 1. Rewrite each of the following using symbols for the logical connectives.

If $x \leq y$ and $y \leq x$ then $x = y$. Ans: $(x \leq y) \wedge y \leq x \rightarrow (x = y)$.

x is odd does not imply x is prime. Ans: $\neg(x$ is odd $\rightarrow x$ is prime$)$.

Note. Some people use the symbol $\sim$ instead of $\neg$.

Vocabulary

Conjunction = and. **Disjunction** = or.

Propostion = sentence.

A proposition of the form $A \rightarrow B$ is called a **conditional** or an **implication**. A is called the **antecedent**, B the **conclusion**.

A proposition that has no connectives is called a **simple proposition**.

Not in book: A proposition that has connectives is called a **compound proposition**.

When we just say 'proposition', we mean it can be either simple or compound.

Example 2. Let p_1 be the prop: x is prime; p_2: x is odd; p_3+ $x = 2$.

Q: Is the following true? $p_1 \rightarrow (p_2 \vee p_3)$. Ans: Yes.

Q: Is the following true? $(p_2 \vee p_3) \rightarrow p_1$. Ans: No.

Q: Is the following true? $(p_2 \vee \rightarrow p_3)$. Ans: This is not a valid or well-formed formula; we can't ask if it's true or false; and we're not interested in it.

The language of Propositional Logic

(We'll see only symbols and formulas today; axioms and rules of inference later.)

Symbols: $p_1, p_2, p_3, \cdots$ (called **propositional variables**); plus the five connectives: $\wedge \vee \neg \rightarrow \leftrightarrow$; plus left and right parentheses: ( ) .

Formulas: First think about this: how would you write some simple rules describing which expressions are or aren't formulas?

We define which expressions are (well-formed) formulas inductively (or recursively), by the following three rules:

F1: Each proposition variable is a formula.

F2: Given two formulas A and B, each of the following is a formula: $(A \wedge B)$, $(A \vee B)$, $\neg A$, $(A \rightarrow B)$, $(A \leftrightarrow B)$.

(F3: Only expressions built by using F1 and F2 are formulas.)

Example 3. Check which of the three props in Example 2 is a formula according to F1 and F2.

Note. For the sake of convenience we: often omit the outermost parentheses; sometimes use [ ] instead of ( ) ; sometimes omit parentheses for consecutive $\vee$'s or $\wedge$'s; use other letters, such as p, q, r.

For example: $(p \vee q \vee r) \leftrightarrow [\neg p \rightarrow (q \vee r)]$. Q: What would be the "strictly correct way" of writing this formula?

Valid or invalid arguments

Example 4. Which of the following are valid arguments?

1. If something is round, then it bounces.
2. Blah is round.
3. $\therefore$ blah bounces.

Ans: Valid. If 1 and 2 are true, then 3 must be true.

1. If something is round, then it bounces.
2. Blah bounces.
3. $\therefore$ blah is round.

Ans: Invalid argument. 1 and 2 do not imply 3.

1. $(p \vee q) \rightarrow r$.
2. $r \rightarrow t$.
3. $\therefore (p \vee q) \rightarrow t$.

Ans: Valid.

1. $p \rightarrow (q \rightarrow r)$.
2. $\neg p \rightarrow t$.
3. $\therefore \neg t \rightarrow (q \vee \neg r)$.

Ans: Valid.

HW # 2, due Wed 31 Jan
Read Section 2.1. Do: p. 53: 1-3.