1. Finish the proof we started in class on Wednesday for "1+1=2."

2. (a) Show (in Predicate Calculus) that $\forall x A(x) \vdash A(y)$.

   (b) Recall that *us* (universal specialization) is not a rule of inference in Predicate Calculus. Use part (a) to explain why *us* can, nevertheless, be used as a *derived* rule of inference.

3. (This question was posed by Jeff.) Does the Compactness Theorem imply that every set Γ of formulas (in Statement Calculus) has a finite subset Δ such that for every truth assignment ϕ, ϕ satisfies Γ iff ϕ satisfies Δ?

(The following exercises are to prepare you for next week's lesson.)

4. Give an algorithm for determining whether or not any given natural number n is the sum of two perfect squares.

   Here's an informal definition of *algorithm*: a detailed and complete set of instructions for someone else to be able to perform with only paper and pencil.

   Your algorithm should give a Yes or No answer after a finite number of steps.

   The "someone else" can be assumed to know how to count and perform basic arithmetic operations on integers.

   Example: For 13, the output of your algorithm should be Yes, since $13 = 2^2 + 3^2$.

Definition A Y/N question is **decidable** if there is an algorithm that, after a finite number of steps, gives a Y or N output to the question. A Y/N question is **semi-decidable** if there is an algorithm that outputs Y in a finite number of steps when the answer is Yes, and produces no output (i.e., never stops) when the answer is No.

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5. Consider the following questions:

   Q1: Is A a theorem of Statement Calculus?

   Q2: Is A a theorem of Predicate Calculus?

   Argue whether each of Q1 and Q2 is decidable, semi-decidable, or neither. (In other words, can you find an algorithm that, for any formula A, tells me in a finite number of steps whether or not A is a theorem?)