

1. Prove or disprove each of the following statements. For each statement, also prove or disprove its converse. (These are all in the context of Statement Calculus, not Predicate Calculus.) Assume Γ is a (possibly infinite) set of formulas.
 - (a) If A is satisfiable, then A is a tautology.
 - (b) If A is consistent, then $\neg A$ is a contradiction. (Be careful: what's the definition of a contradiction?)
 - (c) If Γ is not satisfiable, then it contains a formula that's not satisfiable.
 - (d) If Γ is not satisfiable, then it contains an inconsistent finite subset.
 - (e) If Γ is not consistent, then it contains an unsatisfiable finite subset.
 - (f) Let $\Delta = \{\neg A \mid A \in \Gamma\}$. If Γ is not satisfiable, then Δ is satisfiable.
 - (g) Let $\Delta = \{\neg A \mid A \in \Gamma\}$. If Γ is not consistent, then Δ is consistent.
 - (h) If $\Gamma \models A$, then $\Gamma \cup \{\neg A\}$ is inconsistent.
 - (i) If $\Gamma \models A$, then for some finite subset $\Delta \subset \Gamma$, $\Delta \vdash A$.
2. Prove, without using the Compactness Theorem, that:
 If $\Gamma \vdash A$, then for some finite subset $\Delta \subset \Gamma$, $\Delta \models A$.