These problems are intended to help you review for the final (but they cover only a few topics from the semester). You don't need to turn in any of them; but make sure to do them and ask questions in class. You should know how to do these problems for the final exam.

1. Suppose we pick two vectors $\vec{v}$ and $\vec{w}$ in $\mathbb{R}^2$ "at random" (say each coordinate of each vector is a random integer between 0 and 10).
   (a) Is it more likely that $\vec{v}$ and $\vec{w}$ are parallel or not parallel? (Don't explain.)
   (b) Is it more likely that $\vec{v}$ and $\vec{w}$ are linearly dependent or independent? Why?
   (c) Is it more likely that $\vec{v}$ and $\vec{w}$ form a basis or not for $\mathbb{R}^2$? Why?
2. Suppose we pick three vectors $\vec{u}$, $\vec{v}$ and $\vec{w}$ in $\mathbb{R}^2$ at random.
   (a) Is it more likely that $\vec{u}$, $\vec{v}$ and $\vec{w}$ are linearly dependent or independent? Why?
   (b) Is it more likely that $\vec{u}$, $\vec{v}$ and $\vec{w}$ span or not span $\mathbb{R}^2$? Why?
3. Repeat each of the above questions for $\mathbb{R}^3$, for two, three, and four vectors.
4. Suppose A is an $m \times n$ matrix whose entries are randomly picked integers between say 0 and 100. In each of the following cases, what are all the possible values, and the most likely values, for rank(A), the dimensions of the column space, row space, and nullspace of A, and the number of linearly independent rows and columns in A? Why?
   (a) $m = 3$, $n = 5$
   (b) $m = 7$, $n = 4$
   (c) $m = 3$, $n = 3$. In this case, is $\det(A)$ more likely to be zero or nonzero? Why?
5. Suppose we're given a homogeneous system of m linear equations with n variables: $A\vec{x} = \vec{0}$. In each of the following cases, state whether it's possible or impossible for the system to have zero solutions, one solution, or infinitely many solutions; and state which of the possibilities are more likely if the coefficients are picked at random.
   (a) $m = 3$, $n = 5$
   (b) $m = 7$, $n = 4$
   (c) $m = 3$, $n = 3$.
6. Repeat the above problem for a non-homogeneous system $A\vec{x} = \vec{b}$, where the components of $\vec{b}$ are also random numbers.