Finish example and review theorem from last time.

Review defs: col space, row space, null space.

There is one more subspace that we associate to each matrix:

Definition 1. The **left nullspace** of an $m \times n$ mtx A is the set of all $1 \times m$ row vectors $\vec{y}$ such that $\vec{y}A = \vec{0}$. We write: $\text{LNS}(A) = \{\vec{y} \mid \vec{y}A = \vec{0}\}$. (Equivalent def: $\text{LNS}(A) = \text{NS}(A^T)$.)

Theorem 1. For every matrix A, $\text{CS}(A)$, $\text{RS}(A)$, $\text{NS}(A)$, and $\text{LNS}(A)$ all are vector spaces.

Proof: Exercise (should be able to prove this theorem on your own by now – ask me for help).

Q: For an $m \times n$ matrix A, each is a subspace of $\mathbb{R}^?$ Ans: $\text{CS}(A) \subset \mathbb{R}^m$, $\text{RS}(A) \subset \mathbb{R}^n$, $\text{NS}(A) \subset \mathbb{R}^n$. $\text{LNS}(A) \subset \mathbb{R}^m$.

Review def of dim: both defs, ours and the book's.

Theorem 2. Fundamental Theorem of Linear Algebra, Part I (FTLA): For any $m \times n$ mtx A,

$$\dim(\text{RS}(A)) + \dim(\text{NS}(A)) = n$$

$$\dim(\text{CS}(A)) + \dim(\text{LNS}(A)) = m$$

Proof: next time.

Q: For an arbitrary mtx A, which of the following are guaranteed to equal each other? $\text{RS}(A)$, $\text{CS}(A)$, $\text{NS}(A)$, $\text{LNS}(A)$, $\text{RS}(A^T)$, $\text{CS}(A^T)$, $\text{NS}(A^T)$, $\text{LNS}(A^T)$?

Ans: $\text{RS}(A) = \text{CS}(A^T)$, $\text{RS}(A^T) = \text{CS}(A)$, $\text{NS}(A) = \text{LNS}(A^T)$, $\text{NS}(A^T) = \text{LNS}(A)$. Why?

Example 1. Warm up: effect of lin comb on span.

Let $\vec{v}$ and $\vec{w}$ be two lin indep vectors. Let c be any nonzero scalar.

Q: T or F: $\text{span}(\vec{v}, \vec{w}) = \text{span}(\vec{v}, c\vec{w})$? Ans: T. Why? Need to show two things: 1. Any vector in $\text{span}(\vec{v}, \vec{w})$ is also in $\text{span}(\vec{v}, c\vec{w})$; 2. Any vector in $\text{span}(\vec{v}, c\vec{w})$ is also in $\text{span}(\vec{v}, \vec{w})$.

Q: T or F: $\text{span}(\vec{v}, \vec{w}) = \text{span}(\vec{v}, \vec{v} + \vec{w})$? Ans: T. Why? Need to show two things: ...

So we conclude the following theorem.

Theorem 3. Column operations do not change the column space of a matrix. Row operations do not change the row space of a matrix.

Q: Can row ops change the col space? Yes. Give example.

Q: Can col ops change the row space? Yes. Give example.

Example 2. Warm up: effect of lin comb on lin dep/indep. Let $A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 2 & 6 \end{bmatrix}$.

Q: Are the cols lin dep or indep? Ans: Dep. col3 = col1 + col2.

Q: Do different row ops on A, then answer the above question again. What happens? Ans: After any row op, still have col3 = col1 + col2.

To see this more clearly, replace the numbers by letters!

So we conclude the following theorem.

Theorem 4. Row operations on a matrix do not change linear relationships between the cols. Col operations on a matrix do not change linear relationships between the rows.

HW # 19
Read sec 3.6.
Do: p. 161: 4ab, 13a, 24, 25.

Always prove or explain all your answers, even if the book doesn't ask for it!