
Example 1. Let $\vec{v_1} = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$, $\vec{v_2} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$,

Q: Is $\begin{bmatrix} 3 \\ 3 \\ 4 \end{bmatrix}$ a lin comb of $\vec{v_1}$ and $\vec{v_2}$? Yes. Why? Is $\begin{bmatrix} 2 \\ 3 \\ 4 \end{bmatrix}$ a lin comb of $\vec{v_1}$ and $\vec{v_2}$? No. Why?

We say $\begin{bmatrix} 3 \\ 3 \\ 4 \end{bmatrix}$ is in the *span* of $\vec{v_1}$ and $\vec{v_2}$, but $\begin{bmatrix} 2 \\ 3 \\ 4 \end{bmatrix}$ is not in the *span* of $\vec{v_1}$ and $\vec{v_2}$.

Definition 1. Let $\vec{v_1}, \cdots, \vec{v_n}$ be vectors in $\mathbb{R}^k$. Their **span** is defined as:
$\text{span}\{\vec{v_1}, \cdots, \vec{v_n}\} = \{\vec{w} \mid \vec{w} \text{ is a lin comb of } \vec{v_1}, \cdots, \vec{v_n}\}$ = the set of all vectors that are linear combinations of $\vec{v_1}, \cdots, \vec{v_n}$.

"Span" can be used both as a noun and as a verb.

Example 2. Q: Is (2,3) in the span of $v_1 = (0, 1)$ and $v_2 = (1, 0)$? Ans: Yes. why?

Q: Is $(2, 3) \in \text{span}\{(1, 1), (2, 2)\})$? Ans: No. Why?

Q: Is $(2, 3) \in \text{span}\{(1, 1), (1, 0), (0, 1)\}$? Ans: Yes. Why?

Q: Do $(1, 1)$ and $(2, 2)$ span $\mathbb{R}^2$? No. Why?

Q: Do $(1, 0)$ and $(0, 1)$ span $\mathbb{R}^2$? Yes. Why?

Q: Describe the span of $\{(1, 3)\}$. Ans: The line $y = 3x$ in the xy-plane.

Dimension

Example 3. Let $\vec{v_1} = \begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \end{bmatrix}$, $\vec{v_2} = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}$, $\vec{v_3} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$, $\vec{v_4} = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 1 \end{bmatrix}$.

Q: Is $\begin{bmatrix} 3 \\ 3 \\ 4 \\ 5 \end{bmatrix} \in \text{span}\{\vec{v_1}, \cdots, \vec{v_4}\}$? Yes. Why? Is $\begin{bmatrix} 2 \\ 3 \\ 4 \\ 5 \end{bmatrix} \in \text{span}\{\vec{v_1}, \vec{v_4}\}$? No. Why?

Q: Let $W = \text{span}\{\vec{v_1}, \cdots, \vec{v_4}\}$. Then W is all vectors of the form ...?

Q: Can you find three vectors that span W? Yes. What are they?

Q: Can you find two vectors that span W? No. Proof not necessary right now. We'll see how to prove later.

So we need at least three vectors to span W; we say W has *dimension* 3.

Definition 2. For any vector space V, the **dimension** of V, denoted $\dim(V)$, is the least number of vectors necessary to span V.

Example 4. Q: What is the dimension of $\mathbb{R}^2$? Ans: 2. Why?

Q: What is the dimension of $\mathbb{R}^1$? 1.

Q: Is the line $y = 2x$ a subspace of $\mathbb{R}^2$? Yes.
What is its dimension? 1. Why?

Linear dependence and independence

Definition 3. A set of vectors $\{v_1, v_2, \cdots, v_n\}$ is said to be **linearly dependent** iff at least one of them is equal to a linear combination of the others. A set of vectors $\{v_1, v_2, \cdots, v_n\}$ is said to be **linearly independent** iff none of them is equal to a linear combination of the others.

Example 5. Let $\vec{v_1} = \begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \vec{v_2} = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}, \vec{v_3} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}, \vec{v_4} = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 1 \end{bmatrix}$.

Q: Are the vectors $\vec{v_1}, \cdots, \vec{v_4}$ lin indep? No. Why?

Q:How about the vectors $\vec{v_1}, \vec{v_2}$? Yes. Why?

Q:How about the vectors $\vec{v_1}, \vec{v_2}, \vec{0}$? No. Why?

Note: Most books, including ours, give the following def for lin indep, which looks different but is equivalent to the one above:

Def: A set of vectors $v_1, v_2, \cdots, v_n$ is said to be **linearly dependent** if there exist scalars $c_1, \cdots, c_n$, at least one of which is nonzero, such that $c_1 v_1 + \cdots + c_n v_n = \vec{0}$. A set of vectors $v_1, v_2, \cdots, v_n$ is said to be **linearly independent** if the only time $c_1 v_1 + \cdots + c_n v_n = \vec{0}$ is when all the scalars c_i are 0.

Can you see why these definitions are equivalent?

HW # 17
Read sec 3.5.
p. 150: 1, 3, 4, 10, 11. Ch: 23, 38.