

Recall defs: rref; free col, pivot col, rank, nullity.

Today we will study the general case of  $Ax = b$ , i.e., when  $\vec{b} \neq 0$ .

*Definition 1.* In  $A\vec{x} = \vec{b}$ , if  $\vec{b} = \vec{0}$ , we say the system is **homogeneous**. Otherwise we say it is **nonhomogeneous**.

*Example 1.* (a) Find all solutions to the  $A\vec{x} = \vec{b}$ , where  $A = \begin{bmatrix} 1 & 2 \\ 3 & 6 \end{bmatrix}$ , and  $\vec{b} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$ .

Q: Can you find a few vectors such that all the above solutions are lin combs of these few vectors? Yes: the special solutions.

(b) Find all solutions to the equation  $A\vec{x} = \vec{b}$ , where  $A = \begin{bmatrix} 1 & 2 \\ 3 & 6 \end{bmatrix}$ , and  $\vec{b} = \begin{bmatrix} 5 \\ 10 \end{bmatrix}$ .

Q: Can you find a few vectors such that all the above solutions are lin combs of these few vectors? No!

Q: Find a particular sol  $x_{\vec{P}}$  to the nonhomogeneous system.

Q: T or F: If  $x_{\vec{N}}$  is a solution to the homogeneous system, then  $x_{\vec{P}} + x_{\vec{N}}$  is guaranteed to be a solution to the nonhomog sys. Ans: T. Why?

Q: T or F: *Every* sol to the nonhomogeneous system can be written as  $x_{\vec{P}} + x_{\vec{N}}$  for some  $x_{\vec{N}} \in \text{NS}(A)$ . Ans: T. Why?

This works for any nonhomog system: We can obtain all sols to a non-homog system by knowing just one *particular* sol to it, plus the special sols to the homog system!

$$\text{general sol} = \text{particular sol} + \text{lin combs of special sols}$$

Q: Recall For every  $m \times n$  mtx,  $\text{rank} \leq m$  and  $\leq n$ . Why?

*Definition 2.* An  $m \times n$  mtx  $A$  is said to have **full column rank** iff its rank equals its number of columns ( $r=n$ ); i.e., there is a pivot in every col.

$A$  is said to have **full row rank** iff its rank equals its number of rows ( $r=m$ ); i.e., there is a pivot in every row.

Q: T or F: Every homog sys has at least one solution. Ans: T; the trivial sol:  $\vec{0}$ .

Q: T or F: If a homog sys has a nonzero solution, then it has infinitely many solutions. Ans: T. Why? Any multiple of the nonzero sol is also a sol.

*Conclusion:* Every homog system has either one sol or infinitely many.

Q: Circle all possibilities: A nonhomog system that has more unknowns than eqns can have (1) no sols; (2) only one sol; (3) infinitely many sols. Why?

Ans: no sols, or infinitely many (impossible to have only one). Reason:  $n > m$ , so there must be at least one free var. So the homog sys has infinitely many sols. Why? Now, we have two cases: The nonhomog sys  $A\vec{x} = \vec{b}$  either has sols or doesn't have sols.

Case 1:  $A\vec{x} = \vec{b}$  has a sol. Then, the homog sys has infinitely many sols (why?); so  $A\vec{x} = \vec{b}$  also has infinitely many sols; why?

Case 2:  $A\vec{x} = \vec{b}$  has no sols. Then nothing to prove.

Q: Circle all possibilities: A nonhomog system that has as many unknowns as eqns can have (1) no sols; (2) only one sol; (3) infinitely many sols. Why?

Ans: All three can happen. Reason:

Case 1: there are no free vars. Then  $\text{rank} = n$ ; every col is a pivot col. So there is exactly one sol.

Case 2: there are free vars. So the homog sys has infinitely many sols. So like above, either no sols, or infinitely many.

Give examples...

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Q: Circle all possibilities: A nonhomog system that has more eqns than unknowns can have (1) no sols; (2) only one sol; (3) infinitely many sols. Why?

Ans: All three can happen. Give examples.

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**HW # 16**

Read sec 3.4. Preview sec 3.5.

Do: p. 136: 1, 3, 9, 10abd, 13, 27, 34, 36.