

Review the def of det. We say it's a *recursive* def.

Shortcut for 3x3 mtxs: The “wrap-around” method

Recall: Given  $A = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & k \end{bmatrix}$ ,  $|A| = \dots = a(ek - fh) - b(dk - cg) + c(dh - eg)$ .

The “wrap-around” or “Pac-Man” method (optional to learn) is a shortcut for finding det of 3x3 mtx:

Draw picture...  $|A| = aek + bfg + cdh - gec - hfa - kdb$ . (In book: page 229, problem 26.)

Warning: works only for 3x3 mtxs.

Finding the determinant of an upper or lower triangular mtx is easy!

Do 4x4 example...

### More properties

The following properties are useful for computing dets more quickly. But they are also important conceptually.

Q: Let  $A = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & k \end{bmatrix}$ . Let  $B = \begin{bmatrix} 5a & 5b & 5c \\ d & e & f \\ g & h & k \end{bmatrix}$  How does  $\det(B)$  compare to  $\det(A)$ ? Why?

Q: Does the above work just for  $3 \times 3$  mtxs, or for any  $n \times n$  mtx?

Q: Similar question, but with columns...

*Constant Multiple Rule:* If  $B$  is obtained by multiplying one row or one column of a square mtx  $A$  by a constant  $c$ , then  $\det(B) = c \det(A)$ .

*Proof.* (Optional to learn.) Suppose  $B$  is obtained by multiplying the  $i$ th row of  $A$  by  $c$ . Then  $\det(B) = \sum_{j=1}^n (-1)^{i+j} B_{i,j} \det(\hat{B}_{i,j}) = \sum_{j=1}^n (-1)^{i+j} cA_{i,j} \det(\hat{A}_{i,j}) = c \sum_{j=1}^n (-1)^{i+j} A_{i,j} \det(\hat{A}_{i,j}) = c \det(A)$ .

□

*Example 1.* Q: Let  $A = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & k \end{bmatrix}$ . Let  $B = \begin{bmatrix} 5a & 5b & 5c \\ d & e & f \\ 8g & 8h & 8k \end{bmatrix}$  How does  $\det(B)$  compare to  $\det(A)$ ?

Why?

Q: Let  $B = cA$ , where  $A$  is an  $n \times n$  mtx,  $c$  is a constant. Write  $|B|$  in terms of  $c$  and  $|A|$ .

*Example 2.* Let  $A = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & k \end{bmatrix}$ ,  $B = \begin{bmatrix} d & e & f \\ a & b & c \\ g & h & k \end{bmatrix}$ . How does  $\det(B)$  compare to  $\det(A)$ ? Why?

(Expand along first and second rows, but don't expand fully.)

*Switching Rule:* Switching any two rows of a mtx changes its det by a factor of  $-1$ . Switching any two columns of a mtx changes its det by a factor of  $-1$ . (Note: the rows or the columns do not have to be adjacent.)

Proof: Challenge problem.

Example: Find det of a “permuted” identity mtx.

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*Adding Rule:* Adding a multiple of one row to another (or of one col to another) does not change the det.

Proof: Challenge.

Do a 2x2 example...

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Q: Which of the three (elementary) row ops do or do not change dets?

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Q: What can we say about  $|A|$  if two of  $A$ 's rows are the same? A:  $\det(A) = 0$ . Why?

Q: Does the same property hold for identical columns? A: Yes. Why?

Q: Suppose one of the rows of a square mtx  $A$  is a lin comb of its other rows. What can we say about the det of  $A$ ?

Ans:  $\det(A) = 0$ . Why?

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### Determinant and Transpose

*Theorem 1.* For any square mtx  $A$ ,  $\det(A^T) = \det(A)$ .

Proof: By induction (not difficult) – challenge problem.

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### HW #10

Read p. 209-212.

Do p. 213: 10,11,13a,17-19,24-26; Ch: 5.

Always prove or explain all your answers, even if the book doesn't ask for it!