Remarks:

1. Don't look for solutions in the back of the book until you have done your absolute best.
2. I will post solutions to the problems outside my office. Borrow for copying, then return.
3. Top 3 important things in this (and every math) course: 1. Definitions! 2. Definitions! 3. Definitions! Reveiw def of lin comb.

This entire semester revolves around:

Several equations with several unknowns

Q: Suppose I have \$200 in quarters and dimes. I have 1001 coins altogether. Can you find out how many quarters and how many dimes I have? A:

$10d + 25q = 20000$
$d + q = 1001$
eq1 - 10(eq2): $15q = 20000 - 10010 = 9990$, so $q = 666$, $d = 1001 - 666 = 335$

We'll learn shortcuts and a general method for solving such equations starting next time. Today: interested in different ways of looking at and interpreting such equations: as vectors (called column view), or as equations (called row view). We'll also begin learning about matrices.

Example 1. Solve each of the following systems of equations.

(a) $\begin{aligned} 2x - 3y &= 1 \\ -6x + 8y &= 2 \end{aligned}$

(b) $\begin{aligned} 2x - 3y &= 1 \\ -6x + 9y &= 2 \end{aligned}$

(c) $\begin{aligned} 2x - 3y &= 1 \\ -6x + 9y &= -3 \end{aligned}$

Let's graph each of the above systems of equations on the xy-plane. Before doing so, what do you expect to see on the graphs?

Q: The above equations are said to be *linear equations*. Why? A: Because their graphs are lines.

This entire semester is about **systems** of linear equations in several variables. ("System" of eqns just means several equations considered as one bunch.)

Q: What does linear mean when there are more than two variables? (Can no longer say the graph is a line.) A: See definition below.

Definition 1. A polynomial in several variables is said to be **linear** if the exponent of every variable is 1, and each variable is multiplied only by a constant, not by other variables.

Example 2. Write polynomials and other functions, some linear, some not linear.

A different viewpoint

Example 3. The system $\begin{aligned} 2x - 3y &= 1 \\ -6x + 9y &= -3 \end{aligned}$ can be rewritten as: $x \begin{bmatrix} 2 \\ -6 \end{bmatrix} + y \begin{bmatrix} -3 \\ 9 \end{bmatrix} = \begin{bmatrix} 1 \\ -3 \end{bmatrix}$

So we are trying to write the col vec (1,-3) as a lin comb of ? and ?

First viewpoint: called the "row picture" – graph each eqn.
Second viewpoint: called the "column picture" – draw vectors.

Example 4. One dimension higher: 3-dimensional space, xyz-space:

- Recall: the graph of a linear eqn in 3-space is always a plane.

$$\begin{aligned} 2x - 3y + z &= 6 \\ 2x - 3y - z &= 0 \\ -6x + 8y &= 2 \end{aligned}$$

Q: What is the row picture?
Q: What is the col picture?

Matrices

Instead of writing the full equations in the above example, we abbreviate:

$$\begin{bmatrix} 2 & -3 & 1 \\ 2 & -3 & -1 \\ -6 & 8 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 6 \\ 0 \\ 2 \end{bmatrix}$$

$\begin{bmatrix} 2 & -3 & 1 \\ 2 & -3 & -1 \\ -6 & 8 & 0 \end{bmatrix}$ is called the *coefficient matrix.* (Plural of matrix = matrices; dictionary say matrixes is ok too, but mathematicians don't use it. Like: index, indices; vertex, vertices.)

Definition 2. A **matrix** is a table of numbers.

At this point a matrix is just a very useful way of saving on writing. But in a few weeks we'll see that it's more than just a time-saving tool!

Isn't it strange that the variables are in a col vec? Wouldn't a row vec $[x \;\; y \;\; z]$ seem a lot more natural? We'll find out why we use a col vec in a couple of weeks.

If we solve this system, we'll get: $(x, y, z) = (-15, -11, 3)$.

So we have: $\begin{bmatrix} 2 & -3 & 1 \\ 2 & -3 & -1 \\ -6 & 8 & 0 \end{bmatrix} \begin{bmatrix} -15 \\ -11 \\ 3 \end{bmatrix} = \begin{bmatrix} 6 \\ 0 \\ 2 \end{bmatrix}$ This means: $-15(\text{col}\,1) + (-11)(\text{col}\,2) + 3(\text{col}\,3) = (6, 0, 2)$.

Q: Multiplying an $m \times n$ matrix by an n-component column vector yields what? A: an m-component column vector.

Example 5. Let $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$. Let $\vec{v} = \begin{bmatrix} 8 \\ -3 \\ 2.9 \end{bmatrix}$. Then $A\vec{v} =$?

A is called the 3x3 **identity matrix** (formal def later).

Example 6. $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 8 \\ -3 \end{bmatrix} =$?

HW #2, due Mon
Read section 2.1 (ignore dot products and MATLAB for the time being). Preview section 2.2.
Do P. 29: 1,2,3,10,15,16,18,27.