

Remarks:

1. What we do in class is NOT enough for all the homework problems. You need to read the book also.
2. You may find the book's language a bit unclear at first. The more you read, the more you'll get comfortable with it. This is a good book.

Outline:

- What are vectors?
- Adding and subtracting vectors.
- Pictorial representation of vectors.
- Scalars.
- Linear Combinations.

What is a vector?

Roughly speaking, a vector is just a "bunch of numbers"!

More precisely, a vector is an *ordered set of numbers*.

Example 1. $[2 \ 0]$ is a vector; $[0 \ 2]$ is too; these are two different vectors, since order matters.

$[2 \ -5 \ 7.1]$ is a **row vector**; $\begin{bmatrix} 2 \\ -5 \\ 7.1 \end{bmatrix}$ is a **column vector**.

What's the difference between a row vector and a col vector? None for now. Will see the difference later.

Note. To save space, we sometimes write $(4, 0, -8)$ instead of $\begin{bmatrix} 4 \\ 0 \\ -8 \end{bmatrix}$. So $(4, 0, -8)$ is a column vector.

Each number in the vector is called a **component** of the vector.

E.g., what's the second component of the vector $[3 \ 6 \ 0]$? Ans: 6.

Vectors are used to represent many different things!

Example 2. Start from home. Drive 6 miles East, 2 miles North. Represent this by the vector $[6 \ 2]$. Then continue driving 3 miles East, 5 miles South. Represent this by $[3 \ -5]$.

Q: Where are we relative to home? Ans: Add the two vectors: $[6 \ 2] + [3 \ -5] = [9 \ -3]$.

(Draw picture)

- We add vectors component-wise: one component at a time.

Example 3. I have 4 nickels, 3 dimes, and 2 quarters. You give me 3 nickels and 1 dime, and take 1 quarter. So I'm left with: $[4 \ 3 \ 2] + [3 \ 1 \ -1] = [7 \ 4 \ 1]$.

Note. The book uses boldface letters for vectors. It is difficult to *write* in boldface. So instead we'll use "arrow notation" for vectors:

Book: Let $\mathbf{v} = [4 \ 3]$. Let $\mathbf{w} = [5 \ 3]$. Then $\mathbf{v} + \mathbf{w} = ?$

Us: Let $\vec{v} = [4 \ 3]$. $\vec{w} = [5 \ 3]$. Then $\vec{v} + \vec{w} = ?$

Example 4. $[4 \ 2] + [3 \ 1 \ -1] = ?$ Ans: Undefined.

- Vectors of different size can NOT be added to each other.

Multiplying a vector by a number: scalars

What's $5 + 5 + 5 + 5 + 5 + 5 = ?$

What's $[5 \ 3] + [5 \ 3] + [5 \ 3] + [5 \ 3] + [5 \ 3] + [5 \ 3] = ?$

So, what's $6[5 \ 3] = ?$ Ans: $[30 \ 18]$.

Here the number 6 is called a **scalar**. Why? Because if you draw both vectors, $[5 \ 3]$ and $[30 \ 18]$, on two separate xy -planes, they'll have different lengths but the same direction (slope): we're only changing the "scale on our map" to make one vector look like the other.

Subtracting vectors

Example 5. Let $\vec{v} = [4 \ 3]$. $\vec{w} = [5 \ 3]$. Then $\vec{v} - \vec{w} = ?$ Ans: $[-1 \ 0]$.

How can we represent vector subtraction pictorially?

Step 1. Draw \vec{v} .

Step 2. Multiply \vec{w} by -1 .

Step 3. Add $-\vec{w}$ to \vec{v} .

Linear Combinations

Example 6. Find a and b s.t. $a[5 \ 3] + b[3 \ 2] = [0 \ 1]$.

Ans: Solve two equations with two unknowns:

$$5a + 3b = 0$$

$$3a + 2b = 1.$$

We get: $a = -3$, $b = 5$.

So $(-3)[5 \ 3] + (5)[3 \ 2] = [0 \ 1]$. We say $[0 \ 1]$ is a *linear combination* of $[5 \ 3]$ and $[3 \ 2]$.
(Book sometimes just says combination, instead of linear combination.)

Definition 1. Let $\vec{v}_1, \dots, \vec{v}_n$ be vectors. To say a vector \vec{w} is a **linear combination** of $\vec{v}_1, \dots, \vec{v}_n$ means there exist scalars $c_1, \dots, c_n \in \mathbb{R}$ such that $c_1\vec{v}_1 + \dots + c_n\vec{v}_n = \vec{w}$. The numbers c_1, \dots, c_n are called **coefficients**.

Example 7. Is $[5 \ 6 \ 0]$ a linear combination of $[1 \ 0 \ 0]$, $[0 \ 3 \ 0]$, and $[0 \ 0 \ 8]$? Ans: Yes. Use 5, 2, and 0 for coefficients.

Example 8. Is $[5 \ 6 \ 0]$ a linear combination of $[0 \ 1 \ 1]$, $[0 \ 3 \ 0]$, and $[0 \ 0 \ 8]$? Ans: No. Why?

Example 9. What are all possible lin combs of $[1 \ 0]$ and $[0 \ 1]$? Ans: Any vector $[x \ y] \in \mathbb{R}^2$.

Example 10. What are all possible lin combs of $[1 \ 1]$ and $[2 \ 2]$? Ans: All vectors of the form $[a \ a]$. Why?

HW #1, due Fri 7 Aug

Read section 1.1. Skip section 1.2. Preview section 2.1.

Do P. 6: 1,2,5,8,14,15,17,28,29; Do but don't turn in: [4,6]; Challenge Problems (optional, do not turn in, can bring to office hours): 16,18,19,23.