Math 214 – Linear Systems **Lab 3** Name: ______________________________
Spring Term 2000 **Due Date:** Friday, 7 April.

Directions: This computer lab reviews many of the concepts in Chapter 3. The worksheet is to be written on neatly in pencil – using complete sentences when appropriate. **I recommend taking notes on a separate sheet of paper while exploring MatLab and filling in the worksheet after you have made all your observations.**

I. The function `rref(A)` finds the reduced row echelon form (R in Strang's text) of a matrix A.
As you've seen in class, this form of a matrix is very informative in helping us solve $A\vec{x} = \vec{b}$. For the following matrix, use `rref` to help find the pivot variables, free variables, "special" solutions and general solution to $A\vec{x} = \vec{0}$.

$$A = \begin{bmatrix} 1 & 2 & 0 & 1 & 2 \\ -1 & 2 & 4 & -1 & 0 \\ 5 & -2 & -12 & 5 & 4 \\ 4 & 0 & -9 & 4 & 4 \end{bmatrix}$$

$\texttt{rref}(A) =$

$m =$ ______________ $n =$ ______________ $r =$ ______________

Pivot variables: ____________________________ Free variables: ____________________________

"Special" solutions to $A\vec{x} = \vec{0}$ (one for each free variable):

General solution to $A\vec{x} = \vec{0}$ (the linear combination of the "special" solutions):

This is the ____________________________ of A and has dimension ______________.

II. Let A be the same matrix as in Problem 1. `rref` can be used to help find the solution to $A\vec{x} = \vec{b}$ by applying it to the augmented matrix $[A|\vec{b}]$. Do this for the system $A\vec{x} = \vec{b}$ where $\vec{b} = \begin{bmatrix} 19 \\ 8 \\ 5 \\ 13 \end{bmatrix}$.

`rref([A b])` =

From this, what can you say about $\vec{b}$ and the column space of A? What can you say about the system $A\vec{x} = \vec{b}$?

Now try $\vec{b} = \begin{bmatrix} -4 \\ -6 \\ 10 \\ 4 \end{bmatrix}$.

`rref([A b])` =

Find one particular solution to $A\vec{x} = \vec{b}$.

What is the complete solution to $A\vec{x} = \vec{b}$? (Include a brief sentence on how you got this.)

III. *Is the vector* $\vec{b} = (1, 2, 3)$ *in span of* $\vec{u}$, $\vec{v}$, $\vec{w}$, *where* $\vec{u} = (1, 0, -1)$, $\vec{v} = (2, 2, 4)$, *and* $\vec{w} = (3, 4, 5)$? First, reformulate this question as a linear system. Then answer the question with an **explanation** of how you used the system.

IV. *Is the set of vectors* $\vec{u}$, $\vec{v}$, $\vec{w}$ *linearly independent*? Reformulate the question in terms of a question about a matrix. Then answer the question with an **explanation** of how you used the matrix.

V. *Is the set of vectors* $\vec{u}$, $\vec{v}$, $\vec{w}$ *a basis for* $\mathbf{R}^3$? Explain your answer carefully. (You may refer to your answers in III and IV if they help.)