Reminder: 2nd midterm: 17 Oct. Covers sections 2.3, 2.4, 2.5, 2.7 + chapter 5 + sections 3.1 and 3.2. Four or five problems; like last time, first problem will be just definitions.

Review from last time: col spaces, vector spaces

CS(A) = all vectors that are lin combs of cols of A.

Equivalent description: CS(A) = all vectors $\vec{b}$ for which $A\vec{x} = \vec{b}$ has a solution.

Example 1. Find the col space of each of the following matrices.

$A = I_2$ $\quad B = \begin{bmatrix} 1 & 0 & 3 \\ 0 & 1 & 5 \end{bmatrix}$ $\quad C = \begin{bmatrix} 1 & 0 \\ 0 & 0 \\ 1 & 0 \end{bmatrix}$ $\quad D = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 0 \end{bmatrix}$ $\quad E = \begin{bmatrix} 2 & 1 \\ 3 & 2 \end{bmatrix}$ (Hint: 1. Is the det nonzero? 2. Think "$A\vec{x} = \vec{b}$".)

Vector Space: a set (=collection) of vectors that is closed under (1) vector addition, and (2) scalar multiplication.

Closed under vector addition: for every two vectors that we pick from the set, their sum is also in the set.

Closed under scalar multiplication: for every vector that we pick from the set, and for any scalar, their product is in the set.

Q: Is the col space of every mtx a vector space? Yes. Why?

Theorem 1. For every matrix A, CS(A) is closed under vector addition and scalar multiplication. Therefore CS(A) is a vector space.

Definition 1. If V and W are both vector spaces, and if $V \subset W$, then we say V is a **subspace** of W. ("$\subset$" is the subset symbol; it means: W contains all the vectors that are in V, plus more perhaps.)

Example 2. In the above example, which col spaces are subspaces of each other?

Example 3. Let V be the set of all vectors $(x, y) \in \mathbb{R}^2$ s.t. $x + y = 2$. Let W be the set of all vectors $(x, y) \in \mathbb{R}^2$ s.t. $x + y = 0$.

Is V a vector space? How about W? Ans: V no, W yes. Why?

Example 4. (a) Let $M =$ be the set of all vectors in $\mathbb{R}^3$ that are lin combs of $(1, 0, 0)$ and $(0, 0, 1)$.
Can you describe what M looks like? Ans: The xz-plane.
Is M a vector space? Why? Ans: Yes. Check both closure properties.

(b) Let N be the set of all vectors in $\mathbb{R}^3$ that are lin combs of $(1, 1, 0)$ and $(0, 0, 1)$.
Can you describe what N looks like? Ans: Vertical plane containing the line $y = x$ in the xy-plane. It is a vector space.

(c) For each of the above examples, can you find a subspace that is "smaller"?

Generalized vector spaces

Now we'll see the "full" definition for vector spaces; it's more general and abstract than what we've seen, and very useful in many sciences. (We won't work with it much this semester. If you like it, take math 390, Linear Spaces.)

Definition 2. A (linear) **vector space** is a set V of objects (of *any* kind!) called **vectors**, with two operations, called **vector addition** and **scalar multiplication**, that satisfy the following ten properties (see middle of page 107) :

(1) V is closed under vector addition.

(2) V is closed under scalar multiplication.

(3) Addition in commutative.

(4) Addition in associative.

(5) There is a unique additive identity in V.

(6) Each element in V has a unique additive inverse.

(7) The scalar 1 acts as multiplicative identity.

(8) Scalar multiplication is associative: $c_1(c_2\vec{v}) = (c_1c_2)\vec{v}$.

(9) Scalar multiplication is distributive w.r.t. vector addition: $c(\vec{v} + \vec{w}) = c\vec{v} + c\vec{w}$.

(10) Scalar multiplication is distributive w.r.t. scalar addition. $(c_1 + c_2)\vec{v} = c_1\vec{v} + c_2\vec{v}$.

Example 5. Let M be the set of all 2 by 3 matrices. Let's check that M is a vector space. We call each matrix a vector in M.

Q: Is the set of all 2 by 2 matrices a v.s.? Why or why not?

Q: Is the set of all invertible 2 by 2 matrices a v.s.? Why or why not?

Q: Let $P =$ the set of all polynomials of degree 2 or less. Is P a vector space?

For the rest of the semester, we will work only with vector spaces that are subspaces of $\mathbb{R}^n$; but the power of the above definition is that anything we prove in the coming weeks applies to all vector spaces equally well!

HW #13, due Mon 14 Oct
Do: p. 108: 20-23, 26, 27.

Always prove or explain all your answers, even if the book doesn't ask for it!