Review the def of det. We say it's a *recursive* def.

Shortcut for 3x3 mtxs: The "wrap-around" method

Recall: Given $A = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & k \end{bmatrix}$, $|A| = \cdots = a(ek - fh) - b(dk - cg) + c(dh - eg)$.

The "wrap-around" or "Pac-Man" method (optional to learn) is a shortcut for finding det of 3x3 mtx:

Draw picture... $|A| = aek + bfg + cdh - gec - hfa - kdb$. (In book: page 229, problem 26.)

Warning: works only for 3x3 mtxs.

Finding the determinant of an upper or lower triangular mtx is easy!
Do 4x4 example...

More properties

The following properties are useful for computing dets more quickly. But they are also important conceptually.

Q: Let $A = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & k \end{bmatrix}$. Let $B = \begin{bmatrix} 5a & 5b & 5c \\ d & e & f \\ g & h & k \end{bmatrix}$ How does $\det(B)$ compare to $\det(A)$? Why?

Q: Does the above work just for 3×3 mtxs, or for any $n\times n$ mtx?

Q: Similar question, but with columns...

Constant Multiple Rule: If B is obtained by multiplying one row or one column of a square mtx A by a constant c, then $\det(B) = c\det(A)$.

Proof. (Optional to learn.) Suppose B is obtained by multiplying the ith row of A by c. Then $\det(B) = \sum_{j=1}^{n}(-1)^{i+j}B_{i,j}\det(\hat{B}_{i,j}) = \sum_{j=1}^{n}(-1)^{i+j}cA_{i,j}\det(\hat{A}_{i,j}) = c\sum_{j=1}^{n}(-1)^{i+j}A_{i,j}\det(\hat{A}_{i,j}) = c\det(A)$.

□

Example 1. Q: Let $A = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & k \end{bmatrix}$. Let $B = \begin{bmatrix} 5a & 5b & 5c \\ d & e & f \\ 8g & 8h & 8k \end{bmatrix}$ How does $\det(B)$ compare to $\det(A)$? Why?

Q: Let $B = cA$, where A is an $n\times n$ mtx, c is a constant. Write $|B|$ in terms of c and $|A|$.

Example 2. Let $A = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & k \end{bmatrix}$, $B = \begin{bmatrix} d & e & f \\ a & b & c \\ g & h & k \end{bmatrix}$. How does $\det(B)$ compare to $\det(A)$? Why?
(Expand along first and second rows, but don't expand fully.)

Switching Rule: Switching any two rows of a mtx changes its det by a factor of -1. Switching any two columns of a mtx changes its det by a factor of -1. (Note: the rows or the columns do not have to be adjacent.)

Proof: Challenge problem.

Example: Find det of a "permuted" identity mtx.

Adding Rule: Adding a multiple of one row to another (or of one col to another) does not change the det.

Proof: Challenge.

Do a 2x2 example...

Q: Which of the three (elementary) row ops do or do not change dets?

Q: What can we say about $|A|$ if two of A's rows are the same? A: $\det(A) = 0$. Why?

Q: Does the same property hold for identical columns? A: Yes. Why?

Q: Suppose one of the rows of a square mtx A is a lin comb of its other rows. What can we say about the det of A?
Ans: $\det(A) = 0$. Why?

Determinant and Transpose

Theorem 1. For any square mtx A, $\det(A^T) = \det(A)$.

Proof: By induction (not difficult) – challenge problem.

HW #10, due Fri 5 Oct
Read p. 209-212.
Do p. 213: 10,11,13a,17-19,24-26; Ch: 5.

Always prove or explain all your answers, even if the book doesn't ask for it!