

Block Mtxs.

Review defs: $A_{i,j}$;

Outline of sec 2.7:

1. The definition of the transpose of a mtx. 2. Symmetric mtxs. 3. Properties of the transpose.

Note: We are skipping factorization of mtx for the time being.

Informal definition of the transpose of a mtx: Turn every row into a col, and vice versa.

Notation: transpose of A is written as: A^T (A' in Matlab.)

Examples: ...

Definition 1. The **transpose** A^T of a mtx A is defined by: the ij th entry of A^T is the ji th entry of A ; i.e.,

$$(A^T)_{i,j} = A_{j,i}$$

Q: If A is an m by n mtx, then A^T is a ? by ? mtx? Ans: n by m .

Q: What's the transpose of a row mtx? A: a col mtx.

Q: What's the transpose of the transpose of a mtx — $(A^T)^T = ?$ Ans: A itself.

Q: What is the transpose of the zero and the identity mtxs? A: themselves.

Q: Can you think of any other mtxs A s.t. $A^T = A$?

A: Yes. Any mtx that's symmetric w.r.t. its diagonal.

Definition 2. A mtx A is called **symmetric** if $A^T = A$.

Q: If an m by n mtx is symmetric, what can we say about m and n ? A: $m=n$.

Theorem 1. $(AB)^T = B^T A^T$.

Proof is not very hard. But we'll skip it (challenge problem). Find your own proof, or see p. 99.

Corollary 2. $(A^{-1})^T = (A^T)^{-1}$.

Proof. $AA^{-1} = I$ (why?),

so $(AA^{-1})^T = I^T$,

so (by above theorem), $(A^{-1})^T A^T = I$,

so $(A^{-1})^T$ and A^T are inverses of each other,

so the inverse of A^T is $(A^{-1})^T$,

which means $(A^T)^{-1} = (A^{-1})^T$. □

Block Matrices: handwritten notes

HW #7, due Fri 28 Sep

Read sec 2.7, p. 57-58 (up to example 3). Preview sec 2.3.

Do p. 95: 1b,[2],3,4,7bcd; p. 64: 29, 34; Ch: 35.

Always prove or explain all your answers, even if the book doesn't ask for it!