
Closed book. Closed Notes. May only use the Definitions-Axioms-Theorems handout, with no writings on it. 20 points per problem. Please write very legibly.

Please do not write in this area.

1.

2.

3.

1. Prove that any two distinct great circles on a sphere always intersect in two opposite points. (Assume that they intersect in two points; just prove that the points are opposite points).
2. Suppose an isometry of the Euclidean plane sends every point on the y -axis to some point on the x -axis. What are all possibilities for the image of the line $y = x$ under this isometry? More precisely, if the image is given by the equation $y = mx + b$, what are all possible values for m and b ? Support your answer. (Rigorous proof not required; just give a convincing argument that you haven't missed any possibilities.)
3. Prove Theorem 4.3: If A, B, C are not colinear points, and if S is an isometry, then $S(A), S(B), S(C)$ are not colinear.