Final Exam. Math 360, Axiomatic Geometry.
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Name: _____
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Closed book. Closed Notes. You may only use the Definitions-Axioms-Theorems handout, with nothing extra on it. 20 points per problem. Please write very legibly.

1.	2.	3.	4.	**Circle** (a) or (b) →	5 (a) or (b).	6. (a) or (b).

1. Let A and B be distinct points. Prove that $\overrightarrow{AB} \cap \overrightarrow{BA} = \overline{AB}$.

2. Suppose points A, B, and C are not collinear. Let S and T be similarity transformations such that $S(A) = T(A)$, $S(B) = T(B)$, and $S(C) = T(C)$. Prove $S = T$.

3. Prove the triangle inequality in Neutral Geometry. You may assume the following: In a triangle ABC, if $\measuredangle B < \measuredangle C$, then $AC < AB$.

4. Prove Theorem 3.5: In hyperbolic geometry, there exists a triangle with angle sum strictly less than 180°.

5. Do only part (a) or part (b), *not both*.

   (a) i. Prove in Neutral Geometry that the summit angles of a Saccheri quadrilateral are congruent.
   ii. Prove in Neutral Geometry that the summit of a Saccheri quadrilateral is parallel to it's base.

   (b) Let ABC be a triangle, and let M and N be the midpoints of AB and AC respectively. Prove in Neutral Geometry that $MN||BC$. Hint: You may use part (a) above, without proving it.

6. Do only part (a) or part (b), *not both*.

   (a) Let P and Q be distinct points, and let α and β be (possibly equal) positive real numbers less than 180. Fully describe all possibilities for $S = R_Q(\beta) \circ R_P(\alpha)$, by answering each of the following questions (in terms of P, Q, α, β, when appropriate): Which isometries could S be? If it's a rotation, what is its center and angle of rotation? If it's a translation or reflection or glide-reflection, what is the translation vector and/or the line of reflection? Prove your answers. (Hint: decompose into reflections.)

   (b) Let P and Q be distinct points, and let α and β be (possibly equal) positive real numbers less than 180. Fully describe all possibilities for $S = [R_P(\alpha)]^{-1} \circ R_Q(\beta) \circ R_P(\alpha)$, by answering each of the following questions (in terms of P, Q, α, β, when appropriate): Which of the five types of isometry could S be? If it's a rotation, what is its center and angle of rotation? If it's a translation or reflection or glide-reflection, what is the translation vector and/or the line of reflection? Prove your answers. (Hint: Find fixed points of S. You do not need part (a) to do part (b).)