Some of the following is a review from Calculus. You should become comfortable with them.

Definition: Let's say $f(n)$ **dominates** $g(n)$ if $\lim_{n\to\infty} \frac{f(n)}{g(n)} = \pm\infty$ (or, equivalently, $\lim_{n\to\infty} \frac{g(n)}{f(n)} = 0$).

1. Assume $f(n)$ and $g(n)$ both tend to infinity as n tends to infinity. Is there any relation between "f dominates g", "g dominates f", and "f is $O(g)$"? (For example, are two of them equivalent, or does one imply another?) Prove your answer.

2. Prove that every polynomial is dominated by $n^{\log n}$.

3. Sort the following functions in order of dominance.

   $f(n) = n^{\log n}$, $g(n) = 2^n$, $h(n) = n^{\sqrt{n}}$, $j(n) = n^{0.1n}$, $k(n) = 1.1^n$, $l(n) = n^n$, $m(n) = n!$.

   Hints: 1. Use L'Hopital's rule. 2. Use Stirling's approximation: $n! \approx n^n\sqrt{2\pi n}/e^n$, where $r(n) \approx s(n)$ means as n tends to infinity, $r(n)/s(n)$ tends to 1.