1. True or False: The intersection of a countable set with an uncountable set must be countable. Prove your answer.

2. (a) Give an example of a one-to-one map from a denumerable set to an uncountable set. Prove that your map is one-to-one. (But you do not need to prove that the sets in your example are denumerable or uncountable.)
(b) Does there exist a one-to-one map from an uncountable set to a denumerable set? Prove your answer.

3. (a) Let $f : \mathbf{N^2} \to \mathbf{N}$ be defined by $f(x, y) = xy$. Is f 1-1? Is it surjective? Prove your answers.
(b) Let $g : \mathbf{N^2} \to \mathbf{N}$ be defined by $f(x, y) = 2^x 3^y$. Is g 1-1? Is it surjective? Prove your answers.

4. (a) What does Church's Thesis say? Why has it not been proved yet? Could it maybe be proved in the future?
(b) Use Church's Thesis to "prove" the following:
Let $f(x)$ be a partial computable function from $\mathbf{N}$ to $\mathbf{N}$. Define

$$g(x) = \begin{cases} \max\{f(0), f(1), f(2), f(3), \cdots, f(x)\} & \text{if every term is defined} \\ \text{undefined} & \text{otherwise} \end{cases}$$

Then g is computable.

5. Suppose f is a computable partial function from $\mathbf{N}$ to $\mathbf{N}$. Let $P(x)$ be the predicate: "x is in the domain of f." Do you think P is decidable? Explain why or why not.

6. Suppose f is a computable total function from $\mathbf{N}$ to $\mathbf{N}$. Explain (by using Church's Thesis, if necessary) why bounded minimalization, $\mu\{x < b \mid f(x) = 0\}$ is always defined, but unbounded minimalization, $\mu\{x \mid f(x) = 0\}$ is not necessarily always defined.

7. Define a partial function f by: $f(n) = 1$ if there exist n pairs of twin primes; otherwise $f(n)$ is undefined. Is f computable? Support your answer. You may resort to Church's Thesis, if it's relevant.

8. Suppose f is a computable total function from $\mathbf{N}$ to $\mathbf{N}$. Let P be the predicate: "There exists an x such that $f(x) = 5$." Is P decidable? Why? (You may, but don't have to, use Church's Thesis.)

9. Show multiplication, $M(x, y) = xy$, is computable by defining it using recursion, composition of functions, and the addition function $A(x, y) = x + y$.