Midterm #2. Math 450, Models of Computation.
Instructor: Ramin Naimi

Name: ______________________
Wed 4 Apr 2001

Closed book. Closed Notes. 25 points per problem. Please write very legibly.

1. The set of all finite (but arbitrary length) essays (written in English) is obviously not finite. Is it denumerable or uncountable? Prove your answer (informally, but carefully and convincingly).

2. Show multiplication, $M(x, y) = xy$, is computable, by defining it using recursion, composition of functions (in the book it's referred to as substitution), and the addition function $A(x, y) = x + y$.

3. Suppose that f is a total unary computable function. Prove, by Church's Thesis, that the following function h is URM-computable: $h(x) = \begin{cases} 1 & \text{if } x \in \mathrm{Ran}(f) \\ \text{undefined} & \text{otherwise} \end{cases}$.

4. (a) Suppose f is a partial computable function such that for some $x_0 \in \mathbb{N}$, $f(x_0) = 100$. Given a URM program that computes f, is it possible to find an x (which may or may not equal x_0) such that $f(x) = 100$? Support your answer with an informal but convincing argument. You may use Church's Thesis if you wish.

   (b) Suppose f is a total computable function such that for some $x_0 \in \mathbb{N}$, f is maximized; i.e., $\forall x \in \mathbb{N}$, $f(x_0) \geq f(x)$. Given a URM program that computes f, is it possible to find an x (which may or may not equal x_0) that maximizes f? Support your answer with an informal but convincing argument. You may use Church's Thesis if you wish.