

1. Write a URM program for each of the following functions.

(a) $f(x) = x - 5$. (Be careful with where f is undefined.)

(b)

$$f(k) = \begin{cases} 1 & \text{if } x < 3 \\ 2 & \text{if } 3 \leq x < 5 \\ 3 & \text{otherwise} \end{cases}$$

(c) Let P be the following URM program.

1. J(1,2,3)

2. S(1)

3. S(1)

Let $f(x)$ be the function computed by P . Complete each of the following.

(a) $f(0) =$ $f(1) =$ $f(100) =$

(b) $f(x) = \begin{cases} \text{if} \\ \text{otherwise} \end{cases}$

2. Suppose P is a URM program such that $f_P^{(3)}(x, y, z) = xyz$. Then, $f_P^{(1)}(x) = ?$ Explain why.

3. Use the recursion and composition theorems to prove $f(x) = x!$ is computable. You may assume that the functions $M(x, y) = xy$, $A(x, y) = x + y$, and $k(x) = 1 - x$ are computable. (But you may not make any other assumptions.)

4. Use the recursion and composition theorems to prove the predicate “ x is even” is decidable. You may assume that the functions $M(x, y) = xy$, $A(x, y) = x + y$, and $k(x) = 1 - x$ are computable. (But you may not make any other assumptions.)

5. Let f be a partial function from $\mathbf{N}$ to $\mathbf{N}$, and let g be a total function from $\mathbf{N}$ to $\mathbf{N}$.

(a) Is it possible for $f \circ g$ to be a total function? If yes, give an example. If not, prove why not.

(b) Is $g \circ f$ partial or total? Support your claim.