Midterm #1. Math 450, Models of Computation.
Instructor: Ramin Naimi

Name: _____
Mon 26 Feb 2001

Closed book. Closed Notes. 25 points per problem. Please write very legibly.

1. (a) (5 points) Give the definition of a **computable** function.
   (b) (20 points) Show that the following function is computable by devising a program that will compute it.

$$f(x) = \begin{cases} x/3 & \text{if } x \text{ is a multiple of 3} \\ \text{undefined} & \text{otherwise} \end{cases}$$

2. (25 points) Show that LCM(x, y) = the least common multiple of x and y is computable. You may use any of the computable functions on pages 36-37 (see last page), plus any theorems, lemmas, or corollaries (that you recall accurately) from the textbook.

3. (a) (5 points) Give the definition of a **decidable** predicate.
   (b) (20 points) Use the recursion and composition (i.e., substitution) theorems to prove the predicate "x is even" is decidable. You may assume that the functions $M(x, y) = xy$, $A(x, y) = x + y$, and $k(x) = 1 \dot{-} x$ are computable. (But you may not make any other assumptions.)

4. (25 points) Is there a URM program P that, when given any natural number x as input, stops with a 1 in each of the first x registers? Support your answer.