

Definition: We say a certain condition holds **for all sufficiently large** x if $\exists x_0$ such that that condition holds for all $x > x_0$.

Definition: $f(x) = O(g(x))$ iff $\exists C$ such that $|f(x)| < Cg(x)$ for all sufficiently large x .

(read: $f(x)$ is big-oh of $g(x)$)

Definition: Let's say $f(n)$ **dominates** $g(n)$ if $\lim_{n \rightarrow \infty} \frac{f(n)}{g(n)} = \pm\infty$ (or, equivalently, $\lim_{n \rightarrow \infty} \frac{g(n)}{f(n)} = 0$).

1. Assume $f(n)$ and $g(n)$ both tend to infinity as n tends to infinity. Is there any relation between “ f dominates g ”, “ g dominates f ”, and “ f is $O(g)$ ”? (For example, are two of them equivalent, or does one imply another?) Support your answer.
2. True or false: $n^{\log n}$ dominates every polynomial $p(n)$. Support your answer.
3. Sort the following functions in order of dominance.

$f(n) = n^{\log n}$, $g(n) = 2^n$, $h(n) = n^{\sqrt{n}}$, $j(n) = n^{0.1n}$, $k(n) = 1.1^n$, $l(n) = n^n$, $m(n) = n!$.

Hints: 1. Use L'Hopital's rule. 2. Use Stirling's approximation: $n! \sim n^n \sqrt{2\pi n} / e^n$, where $r(n) \sim s(n)$ means as n tends to infinity, $r(n)/s(n)$ tends to 1.