Department of Mathematics
Gateway – Exponents and Logarithms and Function Notation
Help Sheet

This sheet is to provide you with information as you work toward achieving 90% proficiency on this gateway about **exponents and logarithms**, including exponential and logarithmic functions and equations involving exponents and logarithms. It also deals with **function notation**. As you look through the key ideas below, try to create a realistic picture of what you understand and what you don't.

1. & 2. *Simplifying expressions involving exponents.* When only one base $a > 0$ is involved, the key rules are the following: For all r and s, (i) $a^r a^s = a^{r+s}$, (ii) $(a^r)^s = a^{rs}$ and (iii) $\frac{a^r}{a^s} = a^{r-s}$ and the facts that $a^0 = 1$. When more than one base is involved, additional rules come into play, namely (iv) $(ab)^r = a^r b^r$ and (v) $\left(\frac{a}{b}\right)^r = \frac{a^r}{b^r}$. In addition, you should be able to translate the radical notation $\sqrt[5]{a}$ into the exponential notation $a^{\frac{1}{5}}$. Use these rules to simplify the following expressions so that they contain no negative exponents.

a. $x^2 x^{-5}$ b. $(xy^{\frac{1}{2}})^2(y^{-1}x^3)$ c. $(3x^4y^{-2})(x^{-4}y^{-3})$

d. $\sqrt[3]{x^3y^{-2}z}$ e. $\frac{xyz^{-2}}{x^{-2}yz^2}$ f. $\left(\frac{xy^{-2}}{xy^2}\right)^{-3}$

3. & 4. *Simplifying expressions involving logarithms.* When only one base a is involved, the key rules are the following: For all r and s, (i) $\log_a(rs) = \log_a(r) + \log_a(s)$, (ii) $\log_a\left(\frac{r}{s}\right) = \log_a(r) - \log_a(s)$and (iii) $\log_a(r^s) = s\log_a(r)$ and the fact that $\log_a(a) = 1$. For two special bases, we have special notation: $\log(r) = \log_{10}(r)$ and $\ln(r) = \log_e(r)$. Use these rules to simplify the following expressions so that they contain no products or quotients or exponents. (Some of them may already be completely simplified!)

a. $\log_3(x^2y)$ b. $\log_{10}\left((x^2 - x)^3\right)$ c. $\ln(x + y)$

d. $\log_{12}\left(\frac{x^2}{y^3}\right)$ e. $\frac{\log(x)}{\log(y)}$ f. $\ln\left(\left(\frac{xy^{-2}}{xy^2}\right)^{-3}\right)$

5. & 6. *Solving equations involving exponents and logarithms.* In solving an equation with exponents and logarithms, such as $2^{4x} = 10$ or $\log_3(x) = -2$, you have to use the fact that for the base $a > 0$, the functions $y = a^x$ and $y = log_a(x)$ are inverse functions of each other, i.e. if we take the logarithm, base a of a^x, we get $\log_a(a^x) = x$ and if we raise a to the power $\log_a(x)$, we get $a^{\log_a(x)} = x$. Also helpful will be the fact that for any $a > 0$, with $a \neq 1$, $\log_a(x) = \frac{\log_b(x)}{\log_b(a)}$, for any convenient base $b > 0$, such as 10 or e. Solve the following equations for x.

a. $2^{x-3} = 64$ b. $4^{2x-3} = 32$ c. $5^{x+5} = \frac{1}{3}$

d. $\log_3(x + 7) = -1$ e. $\log_{64}(x^2) = \frac{1}{3}$ f. $\ln(x^2) = 2$

g. $\log(x) + \log(x^2) = -1$ h. $\log_2(x^2 - 1) = 2$ i. $8 + 4^{x-1} = 10$

j. $6^{2x-3} - 5 = 12$ k. $5^{2x} = 6^{3x}$ l. $\log(x) + \log(x - 2) = -1$

7. & 8. *Function notation.* You should be able to evaluate a function at various values and at expressions, e.g., for the function $f(x) = \frac{1}{x^3}$, we find that $f(-3) = \frac{1}{3^3} = \frac{1}{27}$ and $f\left(\frac{1}{\sqrt{x}}\right) = \frac{1}{\left(\frac{1}{\sqrt{x}}\right)^3} = \frac{1}{\frac{1}{x^{\frac{3}{2}}}} = x^{\frac{3}{2}}$. For each of the functions below, evaluate it at the given value and at the given expression.

a. Function: $g(x) = 3^{x+1}$ $\quad g(-2) =$ $\quad g(x+y) =$ $\quad \dfrac{g(x+\Delta x) - g(x)}{\Delta x} =$

b. Function: $h(x) = \ln(x^2+1)$ $\quad h(-2) =$ $\quad h(x+y) =$ $\quad \dfrac{h(x+\Delta x) - h(x)}{\Delta x} =$

c. Function: $j(x) = \log(-x) - \log(x^2)$ $\quad j(-2) =$ $\quad j(x+y) =$ $\quad \dfrac{j(x+\Delta x) - j(x)}{\Delta x} =$

9. *Finding composite functions.* Given the formulas for two functions f and g, find an expression for either $(f \circ g)(x) = f(g(x))$ or $(g \circ f)(x) = g(f(x))$. For example, consider $f(x) = \frac{1}{x}$ and $g(x) = x^2$. Find both $f(g(x))$ and $g(f(x))$. Here are other practice problems:

a. $f(x) = x^3 - x$ $\quad g(x) = \sqrt{x}$ $\quad f(g(x)) =$ $\quad g(f(x)) =$

b. $f(x) = 4^{x-3}$ $\quad g(x) = x^2 - 1$ $\quad f(g(x)) =$ $\quad g(f(x)) =$

c. $f(x) = \log_3(x^2 + x + 1)$ $\quad g(x) = 3^x$ $\quad f(g(x)) =$ $\quad g(f(x)) =$

10. *Sketching the graph of an exponential or logarithmic function.*

You will need to be able to sketch the graphs of elementary exponential and logarithmic functions. For example, try graphing the exponential functions $y = 2^x$, $y = 10^x$ and $y = 5^x$ and the logarithmic functions $y = \log_2(x)$, $y = \log(x)$ and $y = \log_5(x)$. Learn the general shape of these graphs of these functions and be able to label a few key points. For example, we have included the graphs of $y = e^x$ and $y = \ln(x)$ on the graph to the right. Notice which points we have chosen to label.