Occidental College Department of Mathematics

Gateway – Functions

Help Sheet

This sheet is to provide you with further information as you work toward achieving 90% proficiency on this gateway about **functions**. As you look through the key ideas below, try to create a realistic picture of what you understand and what you don't — the first attempt at the gateway should help you with this. While preparing for the second attempt, you should take full advantage of working with your peers, seeking help from other students (both in this course and others), seeing the peer counselors and professional staff at the Center for Teaching and Learning, and talking with your professors.

1.-3. The first three involve evaluating a function at various values and to evaluate functions within algebraic expressions, e.g. $f(x) = \frac{1}{x^3}$.

1. Evaluating a function at a specific number, e.g. find $f(-3)$.
2. Evaluating a function at an algebraic expression, e.g. find $f\left(\frac{1}{\sqrt{x}}\right)$.
3. Setting up the difference quotient used in the limit definition of the derivative, e.g. find

$$\frac{f(x+\Delta x) - f(x)}{\Delta x}$$

4. The composition of functions. Given the formulas for two functions f and g, find an expression for either $(f \circ g)(x) = f(g(x))$ or $(g \circ f)(x) = g(f(x))$. For example, consider $f(x) = \frac{1}{x}$ and $g(x) = x^2$. Find both $f(g(x))$ and $g(f(x))$.

5. The decomposition of functions. Given a function, find two new functions which, when composed, give the original function, e.g. find functions $f(x)$ and $g(x)$ so that $h(x) = (x^2 - x)^3$ is the composition $(g \circ f)(x)$.

6. Translating a written expression involving functions into a algebraic expression, e.g. write an expression for the following statement: The function Q is the sum of the squares of the functions j and k.

7. For this question, formulas for three or four functions are given, e.g. $f(x) = x^2, g(x) = \frac{1}{x}, h(x) = (x-3)^2$ and $k(x) = \sqrt{x}$. Find a formula for the given algebraic combination of the functions. For example, find the formula for $\frac{f+h}{g}(x)$.

8. Using a table of function values, evaluate an algebraic combination or composition of functions at specific numbers, e.g. for the table below, evaluate $\left(\frac{f+g}{fg}\right)(0)$.

x	$f(x)$	$g(x)$
-2	3	17
-1	2	2
0	1	8
1	0	5
2	-1	3

9. Find the domain of a function, e.g. find the domain of the function $w(x) = \frac{1}{\sqrt{2x-1}}$.

10. Given the function P of two variables and a constraint equation relating the two variable, reduce the function P to a function of one variable, e.g. if $P(x, y) = xy - y^2$ and $x^2 = \frac{1}{y}$, find $P(x)$.