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Managing Editor's Note: For years, Andy Vazsonyi has brought a warm, philosophical, wry spirit to his musings on decision sciences. "The Real-Life Adventures of a Decision Scientist," is by no means a new column, but merely a new direction for a column that addresses the age-old question: "What does a decision scientist do?" and "How does he/she do it?"

Which Door Has the Cadillac?

Andrew Vazsonyi, Feature Editor

"No theory is good except on condition that one use it to go on beyond."
André Gide (1869–1951)

It was one of the those beautiful, windless days on the Pacific. Blue skies and big waves from California all the way to Japan. Dick, my daughter's father-in-law, and I stroll along the beach, near Goat Rock. During our long walk, I practice my "tee-ball" game, which involves hitting a golf ball off an extra long, 2 ½-inch tee on every shot along my coastal sand trap. Golf partners usually object to my constant use of the tee during regulation play, so the game is best enjoyed alone.

Today I wasn't doing so well, because I was doing too many things at once. Like keeping an eye out for riptides. The Goat Rock area is notorious for swallowing up people in sleeper waves. I estimated the probability of being washed away at 1/100,000, because two to three people were drowned each year. Deeper down, I wondered if this was a good example of catastrophe theory, when another thought crossed my mind: Why do so many of my golf balls (at 50 cents a piece) fall into the ocean?

Then my companion Dick interrupted my tee-ball practice and calculations with the problem of the Cadillac and two goats. The story is about a TV game show in which you choose from three closed doors and win whatever is behind it. Behind one door is a Cadillac, behind the other two are goats. Suppose you choose door #1, which leads to the car. The game show host, who knows where the car is, first opens door #3, revealing a goat. Then he invites you to switch your guess if you so wish. *Should you switch to door #2?*

After Dick posed the problem, I realized once again why I dislike puzzles so much. One, it takes too long for me to understand them. Two, I'm no good at them. The main reason, however, is that I'm frustrated that Dick believes I should be able to solve this problem because I'm a mathematician. Still, there was something about the problem that tickled my interest.

My first instinct was to say: "Why, it makes no difference! The chances are the same that the car is behind any of the doors, namely one in three."

Immediately the image of the theologian Bayes flashed through my mind, so I thought better of my initial reaction and said simply: "I don't know."

I had no reason to believe that the car was behind any particular door. But after the host opens one door, I have more information, which means I may need to revise my view of the future. Then it struck me that this may be an excellent problem for illustrating decision sciences. New information obviously influences our decisions. Why else would we be looking for information?

As we drive home in my car, which happens to be a Cadillac, I think to myself that I must develop a decision tree to work the problem. I'll assume that the game show Cadillac is behind Door #1. Then suppose I guess Door #1. This is the first branch of my tree. The host will not open Door #1, but he may open Door #2 or #3. Suppose he opens #2, and I switch to Door #3, then I lose. Not good.

"Red light!" Dick says, bringing me back to the other reality.

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I jam on the brakes, the anti-lock system is activated, and the seatbelts keep us from colliding with the dashboard. Then we settle down, continue on our way, and I return to my world of math.

I visualize the second branch of the tree. I guess Door #2, and the host reveals #3. What if I switch to #1 and win? Much better. Now suppose I guess #3, the third branch of the tree. The host opens #2, I guess #1 and win. I can't believe it. Two of the three branches of the tree lead to winning. So the probability is 2/3 if I switch. Of course, I can't believe such nonsense, so I keep mum about it.

I return to the so-called real world, and we manage to arrive safely at home after only a few more near-accidents. Now Dick and his son borrow 3x5 cards and try simulating the problem. But this is such a pain that they don't get anywhere. Also the women in the family want to clear the table for dinner-they're tired of hearing about the Cadillac and goats.

"By Golly, I'm Right!"

I check my calculations on a piece of paper. By golly, I'm right! The probability of winning the Cadillac jumps from 1/3 to 2/3 if I do switch. It makes no sense, but Newton's law makes no sense either. Who could believe such nonsense that the earth affected the moon from such a distance. I feel as if I've slain the dragon, and proceed to jump up and down and dance around (in my mind) like Piglet in Winnie the Pooh.

The next day I call Dick. He's really impressed, because in the meantime he has done his research and found the conclusion.

"How come you're so smart?" he says.

I accept the compliment, but do not tell him that any of the students in my undergraduate decision analysis course could have gotten the same answer. (But would they truly believe it? Yes, if they want to pass the course.) As Descartes implied, the point is not being smarter but to have better methods. I know the earth is round not because I am smarter than Pythagoras, but because I have better information.

I start to tell Dick about my decision tree approach. Never one to mince words, he says, "Cut the math bullshit, and just tell me why."

Thus Spoke the Genius: Paul Erdös

I got even more disturbed when I told the problem to the late Paul Erdös, one of the most famous mathematicians of the century, when he visited my home in 1995.

Erdös was considered by number theorists as one of the greatest experts in probability theory. In a conversation about the use of probability theory in decision making, I mentioned the goats and Cadillac problem and the answer to Erdös, fulling expecting us to move onto the next subject. But, to my surprise, Erdös said, "No, that is impossible, it should make no difference."

I mentioned Bayes, and showed Erdös the decision tree solution I used in my undergraduate course. I reminded him that probability is not a fixed, static thing; it changes as time goes by. To my amazement this didn't convince him. He wanted a straightforward explanation with no decision trees. I gave up at this point, because I have no common sense explanation. I came to the conclusion that unless your are educated in using decision trees, and know how to apply the real-world Bayes theorem, it is hopeless to understand the solution.

According to Richard P. Feynman, the answer to most questions is easy: "I don't know."

So I told Erdös, "You don't know about decision trees so you can't understand the solution. Put on your earphones, listen to your music, and stop bothering me." (When Erdös appeared in my house, the first thing he did was unpack his radio and start listening to classical music. The radio blasted from 5:00 am to midnight. He didn't seem to be able to live without it.)

An hour later Erdös came back really irritated. "What's the matter with you? Why aren't you telling me the reason why I should switch?"

I said that I was sorry, but I didn't have a common sense explanation and only the decision tree analysis convinces me. As a last resort, however, I tried the visual, simulation approach I developed on my computer.

The Monte Carlo method was well known to Erdös because it was first introduced by his good friend and collaborator, Stanislaw Ulam (1909-1984), a mathematician who played a major role in the development of the hydrogen bomb at Los Alamos. The modern method is to use a visual representation, so a decision maker can see what is happening. On the screen I flashed pictures of a sequence of scenarios. The first page showed me guessing door #1. The smiling host opened door #2, and showed a goat happily munching oats. After staying with my original guess, the smiling host opened the door and showed the other happy goat.

In the second scenario, the same events transpired but I won the Cadillac, and the host is crying because he has to pay for it out of his salary. Scenario after scenario followed, and the computer kept running totals of smiling and crying hosts. I ran the program, without the pictures, 100,000 times and found that *if I do not switch, the host will smile about 2/3 of the cases. But if I do switch, he will be crying 2/3 of the cases.*

Erdös objected that he still did not understand the reason why, but was reluctantly convinced that I was right. A few days after he left, he telephoned to say that Ron Graham of AT&T explained to him the reasoning behind the answer and that now he understood. He proceeded to tell me the reasoning but I couldn't fathom his explanation.

Marilyn Knows Best

Later, I got more insight into this by reading the article, "Nation's Mathematicians Guilty of 'Innumeracy,'" in the SKEPTICAL INQUIRER (Vol. 15, Summer 1991, pp. 342-345.) The problem was submitted to Marilyn vos Savant in her magazine column. She answered the question of "to switch or not to switch" by saying, "Yes, you should switch," and gave her explanation. In a later column she published signed letters from 4 PhDs (some quite nasty and sardonic) which severely chastised her for misleading and corrupting the public. Later she gave an alternate explanation. In a still later column she published letters from another 5 PhDs, who all called her an idiot.

The fact is, this problem appeared numerous times not only in the popular press, but in technical and statistical articles and textbooks as well. The problem has caused heated and emotional debates wherever it appeared with many people defending the "obvious" answer that logically there was no point to switch. The common sense answer has been defended not only by plain citizens, but also by high-powered statisticians. I got a big kick out of all this, because in everyday life we change our views all the time in light of new information.

But I was still baffled about the goat controversy. How could somebody like Erdös never have heard of decision trees, which I consider to be the bridge between the world of pure math and the real-world? So I decided to do some research. I looked in my math books. Curiously, I couldn't find anything about decision trees. Then I went to my stat books. Most had absolutely nothing, and the remaining few contained scant information only as an afterthought. Bayes' theorem they had, but only as an incomprehensible formula that no one could understand, much less use. No wonder that the PhD statisticians were getting on vos Savant's case. This was not common knowledge even among the mathematically sophisticated. Unbelievable! I decided that it was my duty to release this long hidden secret from the vaults of Bayes' theorem.

Recently, I got a phone call from Dick. "I have a good solution to the Cadillac problem. You assume that there is a box around the first two doors, and blah, blah, blah."

"I don't get it," I said.

I'm just not interested in ad hoc solutions invented by clever people. I want a method that works for lots of problems. I'm not looking for hundreds of keys to solve these problems. I'm looking for the skeleton key that opens many doors. ■